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Metric Constrained Interpolation, Commutant Lifting and Systems

C. Foias
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PREFACE

This book presents a unified approach for solving both stationary and nonstationary interpolation problems, in finite or infinite dimensions, based on the commutant lifting theorem from operator theory and the state space method from mathematical system theory.

Initially the authors planned a number of papers treating nonstationary interpolation problems of Nevanlinna-Pick and Nehari type by reducing these nonstationary problems to stationary ones for operator-valued functions with operator arguments and using classical commutant lifting techniques. This reduction method required us to review and further develop the classical results for the stationary problems in this more general framework. Here the system theory turned out to be very useful for setting up the problems and for providing natural state space formulas for describing the solutions. In this way our work involved us in a much wider program than original planned. The final results of our efforts are presented here.

The financial support in 1994 from the "NWO-stimulansprogramma" for the Thomas Stieltjes Institute for Mathematics in the Netherlands enabled us to start the research which lead to the present book. We also gratefully acknowledge the support from our home institutions: Indiana University at Bloomington, Purdue University at West Lafayette, Tel-Aviv University, and the Vrije Universiteit at Amsterdam. We warmly thank Dr. A.L. Sakhnovich for his carefully reading of a large part of the manuscript. Finally, Sharon Wise prepared very efficiently and with great care the troff file of this manuscript; we are grateful for her excellent typing.

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The authors

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