MATH221

quiz #3, 11/10/15

Total 100

Show all work legibly. **Name**:_____

1. (20) In the vector space V of all real functions find a basis for span $\{\sin\ t,\ \sin\ 2t,\ \sin\ t\cos\ t\}$.

A basis is:

2. (20) Define $T : \mathbf{P}_2 \to \mathbf{R}^2$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}'(1) \end{bmatrix}$.

(a) (10) Describe Null $T = \{\mathbf{p} : \mathbf{p}(0) = 0, \text{ and } \mathbf{p}'(1) = 0\}$.

Null $T =$

(b) (10) Describe range of T.

Range of T is

3. (20) Suppose $\mathbf{R}^4 = \text{Span}\,\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$. True or False? The vector set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.

Mark one and explain.

▫ True ▫ False

4. (20) Let $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ be a set of vectors in a vector space V so that every $\mathbf{v} \in \mathbf{V}$ has a unique representation as a linear combination of elements of $\mathcal{B}$. True or False? The vector set $\mathcal{B}$ is linearly independent.

Mark one and explain.

▫ True ▫ False

5. (20) Let H be a subspace of V, and $T : V \to W$ is a linear transformation between vector spaces V and W.

   (a) (10) True or False? $T(H)$, the set of images of vectors in H, is a subspace of W.

Mark one and explain.

□ True □ False

(b) (10) True or False? $\dim T(H) \leq \dim H$.

Mark one and explain.

□ True □ False

6. (20) Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. Find rank $\mathbf{u}\mathbf{v}^T$.

rank $\mathbf{u}\mathbf{v}^T =$