Name:

MATH221
test #1, 09/29/16
Sections 1.1–1.7
Solutions
Total 100

Show all work legibly.

1. (25) Solve the system:

$$\begin{array}{rcrcrcr} -x_1 & + & 5x_2 & + & 9x_3 & = & -9 \\ & & 2x_2 & - & 8x_3 & = & 8 \\ x_1 & - & 2x_2 & + & x_3 & = & -1 \end{array}$$

Solution.

$$\begin{bmatrix} -1 & 5 & 9 & -9 \\ 0 & 2 & -8 & 8 \\ 1 & -2 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & -9 & 9 \\ 0 & 1 & -4 & 4 \\ 0 & 3 & 10 & -10 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & -9 & 9 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 22 & -22 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & -5 & -9 & 9 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & -9 & 9 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$x_1 =$ $x_2 =$ $x_3 =$

2. (25) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] = \begin{bmatrix} -1 & 5 & 9 \\ 0 & 2 & -8 \\ 1 & -2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -9 \\ 8 \\ -1 \end{bmatrix}$ True or False? $\mathbf{b}$ is in the set of all linear combinations of the columns of A.

Solution. $\mathbf{b} = 0\mathbf{a}_1 + 0\mathbf{a}_2 + (-1)\mathbf{a}_3$.

Mark one and explain.

▫ True ▫ False

3. (25) Let $\mathbf{a}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 5 \\ 2 \\ -2 \end{bmatrix}$, and $\mathbf{a}_3 = \begin{bmatrix} 9 \\ -8 \\ 1 \end{bmatrix}$. True or False? The vectors $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ are linearly independent.

Solution. The reduced row echelon form of the matrix $\begin{bmatrix} -1 & 5 & 9 \\ 0 & 2 & -8 \\ 1 & -2 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

4. (25) Let A be a 2×3 matrix, and $\mathbf{v}_1$ and $\mathbf{v}_2$ are vectors with three entries so that

$$A\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ and } A\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

True or False? The system of equations $A\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is consistent.

Solution.

$$A\left(2\mathbf{v}_1 + 3\mathbf{v}_2\right) = 2A\mathbf{v}_1 + 3A\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Mark one and explain.

▫ True ▫ False