Name:

MATH221

test #1, 10/27/16

Sections 1.8–1.9, 2.1-2.3 Solutions

Total 100

Show all work legibly.

1. (20) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find A^{-1} if exists.

   Solution.
   $$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$
   $A^{-1} = A$.

2. (20) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. If B is a 2×3 matrix so that $AB = C = \begin{bmatrix} 6 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$. Find B.

   Solution.
   $$B = A^{-1}C = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 1 & 2 \end{bmatrix}$$
   $B =$

3. (20) Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$. Identify all 2×3 matrices X that solve $AX = \begin{bmatrix} 4 & 5 & 6 \\ 4 & 5 & 6 \end{bmatrix}$.

   Solution.
   $$\begin{bmatrix} 0 & 1 & 1 & 2 & 3 \\ 0 & 1 & 4 & 5 & 6 \end{bmatrix} \to \begin{bmatrix} 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
   Hence $X = \begin{bmatrix} x_1 & x_2 & x_3 \\ 4 & 5 & 6 \end{bmatrix}$, x_1, x_2, x_3 are real numbers.

4. (40) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation so that $T(\mathbf{e}_1) = \mathbf{e}_2$, and $T(\mathbf{e}_2) = \mathbf{e}_1$.

   (a) (10) Find A the standard matrix of the transformation.
   Solution.
   $$A = [T(\mathbf{e}_1), T(\mathbf{e}_2)] = [\mathbf{e}_2, \mathbf{e}_1] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

   (b) (15) True or False? T is one-to-one.
   Solution. Since A is invertible T is one-to-one.
   Mark one and explain.
   ▫ True ▫ False

   (c) (15) True or False? T is onto.
   Solution. Since A is invertible T is onto.
   Mark one and explain.
   ▫ True ▫ False