Name:

MATH221

test #3, 12/1/16

Sections 4.1-4.6

Solutions

Total 100

Show all work legibly.

1. (25) Let T be a linear transformation from $\mathbf{P}_2$ to $\mathbf{R}^2$ defined by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(1) \end{bmatrix}$.

   Find A the standard matrix of the transformation (the standard basis for $\mathbf{P}_2$ is $\{1, \mathbf{x}, \mathbf{x}^2\}$).

   Solution. If $\{\mathbf{p}_1(x),\ \mathbf{p}_2(x),\ \mathbf{p}_3(x)\} = \{1, \mathbf{x}, \mathbf{x}^2\}$ is the standard basis for $\mathbf{P}_2$, then $A = [T(\mathbf{p}_1), T(\mathbf{p}_2), T(\mathbf{p}_3)] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

2. (25) Let A be an $n \times n$ matrix. Consider the set $\mathcal{X}$ of all $n \times n$ matrices that satisfy $AX = 0$. True or False? $\mathcal{X}$ is a vector space.

   Solution.

   (a) Let X_1 and X_2 be $n \times n$ matrices such that $AX_1 = AX_2 = 0$. Note that $A(X_1 + X_2) = AX_1 + AX_2 = 0$.

   (b) If $AX = 0$, and c is a scalar, then $A(cX) = cAX = 0$.

   Mark one and explain.
   ▫ True ▫ False

3. (30) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 5 \end{bmatrix}$.

   (a) (15) Find $\dim \operatorname{Row} A$.

   Solution. A is row equivalent to $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$. The matrix has 3 pivots, hence $\dim \operatorname{Row} A = 3$

   (b) (15) Find $\dim \operatorname{Nul} A$.

   Solution. Since $\dim \operatorname{Row} A + \dim \operatorname{Nul} A = 3$, and $\dim \operatorname{Row} A = 3$ one has $\dim \operatorname{Nul} A = 0$.

4. (20) Consider a two function set $S = \{x, e^x\}$. True or False? S is a linearly independent set.

Solution. Assuming linear dependance we can find two constants c_1, and c_2 so that $f(x) = c_1 x + c_2 e^x = 0$ for each $x \in \mathbf{R}$. Note that $0 = f'(x) = c_1 + c_2 e^x = f''(x) = c_2 e^x$, hence $c_2 = 0$, and also $c_1 = 0$. This contradiction completes the proof.

Mark one and explain.

□ True □ False