MATH 221, Fall 2016 - Homework 10 Solutions

Due Tuesday, November 22

Section 3.1

Page 168, Problem 15:

Using the diagonal product method results in:

$\det A = 3(3)(-1) + (0)(2)(0) + (4)(2)(5) - (0)(3)(4) - (5)(2)(3) - (-1)(2)(0) = -9 + 0 + 40 - 0 - 30 + 0 = 1$

Page 168, Problem 33:

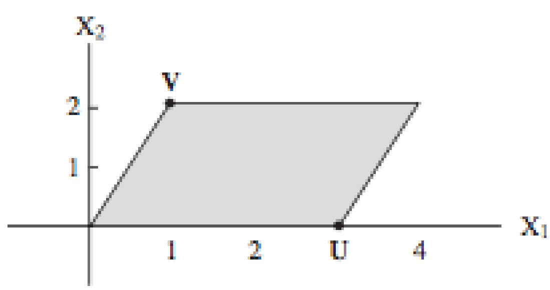
$EA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$, $\det EA = bc - ad$,

$\det E = -1$, $\det A = ad - bc$,

$\det E * \det A = -1 * (ad - bc) = bc - ad$

Page 168, Problem 41:

The graph of the parallelogram is:



The formula for the area of a parallelgoram is $A = bh$, where b is the length of the base and h is height. From this picture, it is clear that the height is 2 and the base is 3, so the area is 6.

The determinant of $[\mathbf{u}\,\mathbf{v}]$ is equal to 6.

If the first entry of $\mathbf{v}$ is changed, the area of the parallelogram is still 6 and the determinant of the matrix is still 6.

Section 3.2

Page 175, Problem 20:

Transforming the matrix $\begin{bmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{bmatrix}$ by $-R_2 + R_1 \to R_1$ yields $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. Because $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$

and the only row-operations on the matrix were adding a multiple of a row, $\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix} = 7$.

Page 175, Problem 21:

Use row-operations to reduce the matrix:

$$\begin{bmatrix} 2 & 3 & 0 \\ 1 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix} : R_3 \leftrightarrow R_1 : \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & 0 \end{bmatrix} : -R_1 + R_2 \to R_2 : \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 3 & 0 \end{bmatrix} : -2R_1 + R_3 \to R_3 : \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -2 \end{bmatrix} :$$

$R_2 + R_3 \to R_3 : \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$. Thus, the determinant is -1 (because of the row-interchange at the beginning).

Because the determinant does not equal 0, the matrix is invertible.

Page 175, Problem 22:

Use row-operations to reduce the matrix:

$$\begin{bmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & 5 & 3 \end{bmatrix} : R_1 \leftrightarrow R_2 : \begin{bmatrix} 1 & -3 & -2 \\ 5 & 0 & -1 \\ 0 & 5 & 3 \end{bmatrix} : -5R_1 + R_2 \to R_2 : \begin{bmatrix} 1 & -3 & -2 \\ 0 & 15 & 9 \\ 0 & 5 & 3 \end{bmatrix} :$$

$-\frac{1}{3}R_2 + R_3 \to R_3 : \begin{bmatrix} 1 & 3 & -2 \\ 0 & 15 & 9 \\ 0 & 0 & 0 \end{bmatrix}$. Because the determinant is equal to 0, the matrix is not invertible.

Page 176, Problem 35:

By Theorem 6, $\det U^T U = \det U^T * \det U$. By Theorem 5, $\det U^T = \det U$, so

$\det U^T U = (\det U)^2$. Since $U^T U = I$, $1 = \det I = \det U^T U = (\det U)^2$. Therefore, $\det U = \pm 1$.

Page 176, Problem 37:

Straightforward calculation of $\det A = 3(1) - 0(6) = 3$ and $\det B = 2(4) - 0(5) = 8$ shows that

$(\det A)(\det B) = 3(8) = 24$. The determinant of the matrix product $AB = \begin{bmatrix} 6 & 0 \\ 17 & 4 \end{bmatrix}$ is

$\det AB = 6(4) - 0(17) = 24$. Thus, $24 = \det AB = (\det A)(\det B) = 24$.

Page 176, Problem 40:

Straightforward calculations using the properties of determinants:

- $\det AB = (\det A)(\det B) = (-1)(2) = -2$
- $\det B^5 = (\det B)^5 = 2^5 = 32$
- $\det 2A = 2^4 \det A = 16(-1) = -16$
  - NOTE: When A is an $n \times n$ matrix, $\det(rA) = r^n \det A$, by factoring an r out of each of the n rows.
- $\det A^T A = (\det A^T)(\det A) = (\det A)(\det A) = -1(-1) = 1$
- $\det B^{-1}AB = (\det B^{-1})(\det A)(\det B) = (\frac{1}{2})(-1)(2) = -1$
  - NOTE: $BB^{-1} = I \Longrightarrow \det BB^{-1} = (\det B)(\det B^{-1}) = \det I = 1$, so $\det B^{-1} = \frac{1}{\det B}$

Page 176, Problem 41:

Calculate each determinant: $\det A = (a+e)(d) - (b+f)(c) = ad + de - bc - cf = ad - bc + ed - fc$,

$\det B = ad - bc$, $\det C = ed - fc$. It is clear then that $\det A = \det B + \det C$