

Homework 10  
Answer key

5.5

①  $\min z = 14$      $B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$      $B^{-1} = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$

a.)  $b^* = B^{-1}b = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \end{bmatrix} = \begin{bmatrix} -7 + 16 \\ -7 + 8 \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \end{bmatrix} \geq 0$

$z = 2(9) - 3(1) + 0 + 0 = \boxed{15} @ (9, 1, 0, 0)$

b.)  $b^* = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix} \geq 0$

$z = 2(8) - 3(0) + 0 + 0 = \boxed{16} @ (8, 0, 0, 0)$

④  $B = \begin{bmatrix} 2 & -6 \\ 3 & -3 \end{bmatrix}$      $B^{-1} = \begin{bmatrix} -1/4 & -1/2 \\ -1/4 & 1/6 \end{bmatrix}$

a.)  $b^* = B^{-1}b = \begin{bmatrix} -1/4 & 1/2 \\ -1/4 & 1/6 \end{bmatrix} \begin{bmatrix} 6 + \lambda \\ 15 \end{bmatrix} = \begin{bmatrix} -6 - 1/4\lambda \\ 1 - 1/4\lambda \end{bmatrix}$

$\begin{cases} -6 - 1/4\lambda \geq 0 & \Rightarrow -24 \geq \lambda \\ 1 - 1/4\lambda \geq 0 & \Rightarrow 4 \geq \lambda \end{cases} \quad \lambda \leq \boxed{-24}$

$\boxed{(1 - 1/4\lambda, 0, 0, 0, -6 - 1/4\lambda)}$

$z = 5(1 - 1/4\lambda) - 2(-6 - 1/4\lambda) + 0 = \boxed{38 - 3/4\lambda}$

b.)  $b^* = \begin{bmatrix} -1/4 & 1/2 \\ -1/4 & 1/6 \end{bmatrix} \begin{bmatrix} 6 \\ 15 + \lambda \end{bmatrix} = \begin{bmatrix} 6 + 1/2\lambda \\ 1/6\lambda \end{bmatrix} \geq 0$

$\begin{cases} 6 + 1/2\lambda \geq 0 & \Rightarrow \lambda \geq -12 \\ 1/6\lambda \geq 0 & \Rightarrow \lambda \geq 0 \end{cases} \quad \lambda \geq \boxed{0}$

$$(1/6\lambda, 0, 0, 0, 6 + 1/2\lambda)$$

$$z = 5(1/6\lambda) - 2(6 + 1/2\lambda) + 21 = \boxed{-1/6\lambda + 9}$$

$$c) b^* = \begin{bmatrix} -1/4 & 1/2 \\ -1/4 & 1/6 \end{bmatrix} \begin{bmatrix} 6 + \lambda \\ 15 + \lambda \end{bmatrix} = \begin{bmatrix} 6 + 1/4\lambda \\ 1 - 1/12\lambda \end{bmatrix} \geq 0$$

$$\left. \begin{array}{l} 6 + 1/4\lambda \geq 0 \Rightarrow \lambda \geq -24 \\ 1 - 1/12\lambda \geq 0 \Rightarrow \lambda \leq 12 \end{array} \right\} \boxed{-24 \leq \lambda \leq 12}$$

$$(1 - 1/12\lambda, 0, 0, 0, 6 + 1/4\lambda)$$

$$\min z = 5(1 - 1/12\lambda) - 2(6 + 1/4\lambda) + 21 = \boxed{14 - 1/12\lambda}$$

$$\textcircled{8} \quad b = \begin{bmatrix} 350 \\ 250 \\ 100 + \lambda \\ 320 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 10 & 11 & 0 \\ 0 & -12 & 6 & 0 \\ 0 & 4 & 3 & 0 \\ 0 & 9 & 7 & 1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 1 & 7/6 & -6 & 0 \\ 0 & 1/4 & -1/2 & 0 \\ 0 & -1/3 & 1 & 0 \\ 0 & 1/12 & -5/2 & 1 \end{bmatrix}$$

$$b^* = \begin{bmatrix} 1 & 7/6 & -6 & 0 \\ 0 & 1/4 & -1/2 & 0 \\ 0 & -1/3 & 1 & 0 \\ 0 & 1/12 & -5/2 & 1 \end{bmatrix} \begin{bmatrix} 350 \\ 250 \\ 100 + \lambda \\ 320 \end{bmatrix} = \begin{bmatrix} 125/3 - 6\lambda \\ 25/2 - \lambda/2 \\ 50/3 + \lambda \\ 545/6 - 5/2\lambda \end{bmatrix} \geq 0$$

$$\Rightarrow \begin{array}{l} 125/18 \geq \lambda \\ 25 \geq \lambda \\ -50/3 \leq \lambda \\ 109/3 \geq \lambda \end{array}$$

$$\boxed{-50/3 \leq \lambda \leq 125/18}$$

Section 5.6

③ Max  $z = 11x_1 + 4x_2 + x_3 + 15x_4$   
 s.t.  $3x_1 + x_2 + 2x_3 + 4x_4 \leq 28$   
 $8x_1 + 2x_2 - x_3 + 7x_4 \leq 50$   
 $x_1, x_2, x_3, x_4 \geq 0$

$B = \begin{bmatrix} 3 & 1 \\ 8 & 2 \end{bmatrix}$       $B^{-1} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$

a)  $b^* = B^{-1}b = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 28 \\ 49 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix} \geq 0$

Optimal solution  $\boxed{(0, 0, 0, 7)}$       $z = 11(0) + 4(0) + 0 + 15(7) = \boxed{105}$

b)  $b^* = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 29 \\ 50 \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \end{bmatrix} \leq 0$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	
$x_4$	-2	0	5	1	2	-1	8
$x_2$	11	1	-18	0	-7	4	-3
	3	0	2	0	2	1	108
$x_4$	$19/18$	$5/18$	0	1	$1/18$	$1/9$	$43/6$
$x_3$	$-11/18$	$-1/18$	1	0	$7/18$	$-2/9$	$1/6$
	$38/9$	$1/9$	0	0	$1/9$	$13/9$	$107\frac{2}{3}$

$\boxed{(0, 0, 11/6, 43/6)}$       $z = 107\frac{2}{3}$

c)  $b^* = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 28 \\ -23 \end{bmatrix} = \begin{bmatrix} 79 \\ -288 \end{bmatrix} \leq 0$

$$-2 + \frac{25}{18}$$

$$-36$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	
$x_4$	-2	0	5	1	2	-1	79
$x_2$	11	1	18	0	-7	4	-288
	3	0	2	0	2	1	33
$x_4$	$19/18$	$5/18$	0	1	$1/18$	$1/9$	-1
$x_3$	$-11/18$	$-1/18$	1	0	$7/18$	$-2/9$	$1/6$
	$38/9$	$1/9$	0	0	$11/9$	$13/9$	1

No feasible solutions

$$⑤ \quad B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$b^* = B^{-1}b = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 8 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

	$x_1$	$x_2$	$x_3$	$x_4$	
$x_1$	1	0	7	4	7
$x_2$	0	1	5	3	-1
	0	0	2	2	17

No feasible solutions

### Section 5.7

$$② \quad a. \text{ optimal } (19, 8, 0, 0) \quad 19 + 8 + 2(0) + 2(0) = 27 \leq 37$$

$$\Rightarrow z = 14 \text{ at } (19, 8, 0, 0)$$

b. Since  $19 + 8 + 0 + 0 = 27 \neq 17$  add slack variable  $x_5$

$$-(x_1 + 7x_3 + 4x_4) = 19$$

$$(x_2 + 5x_3 + 3x_4) = 8$$

$$x_1 + x_2 + 2x_3 + 2x_4 + x_5 = 17$$

$$-10x_2 - 5x_3 + x_5 = -10$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_1$	1	0	7	4	0	19
$x_2$	0	1	5	3	0	8
$x_5$	0	0	-10	-5	1	-10
	0	0	2	2	0	-14
$x_1$	1	0	0	1/2	7/10	12
$x_2$	0	1	0	1/2	1/2	3
$x_3$	0	0	1	1/2	-1/10	1
	0	0	0	1	1/5	-10

min  $z = 16$   
 $\circ (12, 3, 1, 0)$

c) At  $(19, 8, 0, 0)$   $x_1 - x_2 = 19 - 8 = 11 \neq 7$  add slack variable  $x_5$

$$\begin{aligned}
 - (x_1 - 7x_3 + 4x_4) &= 19 \\
 x_2 + 5x_3 + 3x_4 &= 8 \\
 x_1 - x_2 + x_5 &= 7 \\
 \hline
 12x_3 - x_4 + x_5 &= 4
 \end{aligned}$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_1$	1	0	7	4	0	19
$x_2$	0	1	5	3	0	8
$x_5$	0	0	12	-1	1	-4
	0	0	2	2	0	-14
$x_1$	1	0	55	0	4	3
$x_2$	0	1	41	0	3	-4
$x_4$	0	0	-12	1	-1	4
	0	0	26	0	2	-22

no feasible solutions

d)  $(19, 8, 0, 0)$   
 $19 + 2(8) + 0 - 0 = 35 \neq 38$

introduce  $x_5$

$$\begin{aligned}
 - (x_1 - 7x_3 + 4x_4) &= 19 \\
 -2 (x_2 + 5x_3 + 3x_4) &= 8 \\
 x_1 + 2x_2 + 70x_3 - 2x_4 - x_5 &= 35 \Rightarrow -17x_3 + 12x_4 + x_5 = 0 \\
 -1 (17x_3 - 12x_4 - x_5) &= 0
 \end{aligned}$$

minimum  $z = 14$  at  $(19, 8, 0, 0, 0)$

④ a) at  $(8, 18, 0)$

$$10 - 18 - 0 = -2 \neq 12$$

Introduce  $x_7$

$$\begin{aligned} -2(x_1 - 8/5x_3 + 1/5x_4 - 2/5x_5) &= 8 \\ (x_2 + 12/5x_3 + 1/5x_4 + 3/5x_5) &= 18 \\ (13x_3 + 2x_5 + x_6) &= 70 \\ 2x_1 - x_2 - 5x_3 & -x_7 = 12 \end{aligned}$$

$$\begin{aligned} 3/5x_3 - 1/5x_4 + 7/5x_5 & -x_7 = 14 \\ -3/5x_3 + 1/5x_4 - 7/5x_5 & +x_7 = -14 \end{aligned}$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	
$x_1$	1	0	$-8/5$	$1/5$	$-2/5$	0	0	8
$x_2$	0	1	$12/5$	$1/5$	$3/5$	0	0	18
$x_6$	0	0	13	0	2	1	0	70
$x_7$	0	0	$-3/5$	$1/5$	$-7/5$	0	1	-14
	0	0	1	1	1	0	0	70
$x_1$	1	0	$-56/13$	$13/13$	0	0	$-2/7$	12
$x_2$	0	1	$8/13$	$-2/13$	0	0	$3/7$	12
$x_6$	0	0	$9/7$	$-6/7$	0	1	$10/7$	50
$x_5$	0	0	$-1/7$	$3/7$	1	0	$-5/7$	10
	0	0	$8/7$	$1/7$	0	0	$5/7$	60

Max  $z = 60$  @  $(12, 12, 0)$

b) at  $(18, 8, 0)$   $x_3 = 0 \neq 6$

add slack variable  $x_7$

$$\begin{aligned} -1/13(13x_3 + 2x_5 + x_6) &= 70 \\ x_3 & -x_7 = 0 \end{aligned}$$

$$2/13x_5 - 1/13x_6 - x_7 = 8/13$$

$$2/13x_5 + 1/13x_6 + x_7 = -8/13$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	
$x_1$	1	0	$-8/5$	$1/5$	$-2/5$	0	0	8
$x_2$	0	1	$12/5$	$1/5$	$3/5$	0	0	18
$x_6$	0	0	13	0	2	1	0	70
$x_7$	0	0	0	0	$2/13$	$1/13$	1	$-8/13$
	0	0	1	1	1	0	0	70

no feasible solutions

c)  $(8, 18, 0) \Rightarrow 16 + 18 = 34 \neq 39$   
 add slack variable  $x_7$

$$\begin{aligned} -2(x_1 - 8/5x_3 + 1/5x_4 - 2/5x_5) &= 8 \\ -(x_2 + 12/5x_3 + 1/5x_4 + 3/5x_5) &= 18 \\ 2x_1 + x_2 & \quad \quad \quad -x_7 = 39 \end{aligned}$$

$$\begin{aligned} -4/5x_3 - 3/5x_4 + 1/5x_5 - x_7 &= 5 \\ 1/5x_3 + 3/5x_4 - 1/5x_5 + x_7 &= -5 \end{aligned}$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	
$v_1$	1	0	$-8/5$	$1/5$	$-8/5$	0	0	8
$v_2$	0	1	$12/5$	$1/5$	$3/5$	0	0	18
$v_6$	0	0	13	0	2	1	0	70
$v_7$	0	0	$1/5$	$3/5$	$-1/5$	0	1	-5
	0	0	1	1	1	0	0	70
$v_1$	1	0	$-16/5$	1	0	0	-2	18
$v_2$	0	1	$24/5$	2	0	0	3	3
$x_6$	0	0	21	6	0	1	10	20
$v_5$	0	0	-4	-3	1	0	-5	25
	0	0	5	4	0	0	5	45

max  $z = 45$  @  $(18, 3, 0)$

d) at  $(8, 18, 0) \Rightarrow 8 + 18 + 0 = 26 \neq 20$   

$$\begin{aligned} -(x_1 - 8/5x_3 + 1/5x_4 - 2/5x_5) &= 8 \\ -(x_2 + 12/5x_3 + 1/5x_4 + 3/5x_5) &= 18 \\ x_1 + x_2 + x_3 & \quad \quad \quad = 20 \\ 11/5x_3 - 2/5x_4 - 1/5x_5 &= -6 \end{aligned}$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	
$x_1$	1	0	$-\frac{8}{5}$	$\frac{11}{5}$	$-\frac{2}{5}$	0	8
$x_2$	0	1	$\frac{12}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	0	18
$x_6$	0	0	13	0	2	1	70
$x$	0	0	$\frac{11}{5}$	$-\frac{2}{5}$	$-\frac{1}{5}$	0	-6
	0	0	1	1	1	0	70
							$\frac{12}{5} + \frac{3}{5}$
$x_1$	1	0	-2	1	0	0	20
$x_2$	0	1	3	-1	0	0	0
$x_6$	0	0	15	-4	0	1	10
$x_5$	0	0	-1	2	1	0	30
	0	0	2	-1	0	0	40
$x_1$	1	0	$-\frac{3}{2}$	0	$-\frac{1}{2}$	0	5
$x_2$	0	1	$\frac{5}{2}$	0	$\frac{1}{2}$	0	15
$x_6$	0	0	13	0	2	1	70
$x_4$	0	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	15
	0	0	$\frac{3}{2}$	0	$\frac{1}{2}$	0	55

MAX  $Z = 55$  @  $(5, 15, 0)$

$0 \neq 0 = 0 + 18 + 8 \Rightarrow (0, 18, 8)$   
 $(8) = 2x_1 + 15x_2 - 4x_3 + 11x_4 + 3x_5 + 70x_6$   
 $(18) = 12x_1 + 5x_2 + 13x_3 - 2x_4 + 3x_5 + 70x_6$   
 $0 = 11x_1 - 2x_2 + 11x_3 - 2x_4 - x_5 + 70x_6$   
 $0 = 11x_1 - 2x_2 + 11x_3 - 2x_4 - x_5 + 70x_6$