Homework #2
Answer Key

Section 2.4

(2)

a. $x_{11} + x_{12} + x_{13} + x_{14} \leq 600$
$x_{21} + x_{22} + x_{23} + x_{24} \leq 1000$
$x_{11} + x_{21} = 300$
$x_{12} + x_{22} = 350$
$x_{13} + x_{23} = 400$
$x_{14} + x_{24} = 450$
Minimize
$20x_{11} + 16x_{12} + 30x_{13} + 20x_{14} + 45x_{21} + 39x_{22} + 50x_{23} + 44x_{24}$

b. $0 \leq y \leq 1$ $\quad y \geq \frac{1}{300}(x_{11})$
$0 \leq z \leq 1$ $\quad z \geq \frac{1}{350}(x_{12})$
$x_{11} + x_{12} + x_{13} + x_{14} \leq 600$
$x_{21} + x_{22} + x_{23} + x_{24} \leq 1000$
$x_{11} + x_{21} = 300$
$x_{12} + x_{22} = 350$
$x_{13} + x_{23} = 400$
$x_{14} + x_{24} = 450$
minimize:
$20x_{11} + 16x_{12} + 30x_{13} + 20x_{14} + 45x_{21} + 39x_{22} + 50x_{23} + 44x_{24} +$
$50y + 60z$

(6) $x_{11} + x_{12} + x_{13} \leq 80$
$x_{21} + x_{22} + x_{23} \leq 80$
$x_{11} + x_{21} = 50$
$x_{12} + x_{22} = 50$
$x_{13} + x_{23} = 50$
$U + V = x_{21}$

U = # of x_{21} units up to 20
V = # of x_{21} units above 20

minimize:
$8x_{11} + 17x_{12} + 19x_{13} + 21x_{22} + 22x_{23} + 10U + 13V$

Section 2.5

(3) x_i = number of refrigerators bought in month i, $i = 1, 2, 3$
y_i = number of refrigerators sold in month i, $i = 1, 2, 3$
s_i = # of refrigerators stored at end of period i, $i = 1, 2, 3$

$x_i \leq 65$
$y_i \leq 100$
$s_i \leq 45$
$25 + x_1 = y_1 + s_1$
$s_1 + x_2 = y_2 + s_2$
$s_2 + x_3 = y_3 + s_3$
maximize
$90y_1 + 110y_2 + 105y_3 - 60x_1 - 65x_2 - 68x_3$
$-7(s_1 + s_2 + s_3)$

(7) x_i = hours process 1 is used in month i, $i = 1, 2, 3$
y_i = hours process 2 is used in month i, $i = 1, 2$
s_i = # of units stored at end of month i, $i = 1, 2, 3$

$12x_1 + 9y_1 = 300 + s_1$
$s_1 + 12x_2 + 9y_2 = 300 + s_2$
$s_2 + 12x_3 = 300 + s_3$
$6x_i + 2y_i \leq 110 \quad i = 1, 2, 3$
$3x_i + 4y_i \leq 95 \quad i = 1, 2, 3$
minimize
$36.6(x_1 + x_2 + x_3) + 36.2(y_1 + y_2 + y_3) +$
$26.3(x_1 + x_2 + x_3) + 26.4(y_1 + y_2 + y_3) +$
$20(s_1 + s_2 + s_3)$

(10)

A_i = Units of A sold in month i, $i=1,2,3$

B_i = Units of B sold in month i, $i=1,2,3$

AP_i = Units of A produced in month i, $i=1,2,3$

BP_i = Units of B produced in month i, $i=1,2,3$

C_i = Units of C unused at end of month i, $i=1,2,3$

AS_i = Units of A stored at end of month i, $i=1,2,3$

BS_i = Units of B stored at end of month i, $i=1,2,3$

U_i = number of hr, regular time labor for month i, $i=1,2,3$

V_i = number of hr, OT time labor for month i, $i=1,2,3$

For each month $i=1,2,3$

$3AP_i + 2BP_i = U_i + V_i \quad U \le 400 \quad V \le 100$

$AS_i \le 100 \quad BS_i \le 100$

$C_i = C_{i-1} - 5AP_i - 8BP_i \quad C_0 = 3500$

$AS_{i-1} + AP_i = A_i + AS_i \quad AS_0 = 0$

$BS_{i-1} + BP_i = B_i + BS_i \quad BS_0 = 0$

$8AS_i + 9BS_i + 0.8C_i \le 3000$

$$\sum_{i=1}^{3} (5AP_i + 8BP_i) \le 3500$$

Maximize:

$$60A_1 + 62A_2 + 64A_3 + 50B_1 + 58B_2 + 56B_3 - 12(U_1+U_2+U_3) - 18(V_1+V_2+V_3) - 1.5C_3$$

Section 3.1

(2) Set B should be

$$x_1 + x_2 + x_3 = 6$$
$$x_1 + 2x_2 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \geq 0$$

In this case, set A and B would be equal.

(3) a. minimize: $-3x_1 + 2x_2$

S.t.

$$5x_1 + 2x_2 - 3x_3 + x_4 + x_5 = 7$$
$$3x_2 - 4x_3 + x_6 = 6$$
$$x_1 + x_3 - x_4 - x_7 = 11$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

b. minimize $-x_2' + (x_3' - x_3'') + (x_4' - x_4'')$

S.t.

$$x_1 + x_2' - x_5 = 6$$
$$-x_2' + x_3' - x_3'' - x_4' + x_4'' + x_6 = 1$$
$$5x_1 + 6x_2' + 7x_3' - 7x_3'' - 8x_4' + 8x_4'' - x_7 = 2$$

$$x_1, x_2', x_3', x_3'', x_4', x_5'', x_5, x_6, x_7 \geq 0$$

c. minimize $x_1 + x_3 - x_4 + 48$

S.t.

$$-3x_1 + x_2 - x_3 + 2x_4 = -50$$
$$x_1 - x_2 + x_4 + x_5 = 100$$
$$2x_2 - x_3 - x_4 - x_6 = -150$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

d. minimize $-6x_1 + 2x_2' - 2x_2'' - 9x_3 - 300$

s.t. $2x_1 - 6x_2' + 6x_2'' - x_3 + x_4 = 100$

$x_1 + x_2' - x_2'' + 9x_3 + x_5 = 200$

$x_1 + x_6 = 50$

$x_2' - x_2'' - x_7 = -60$

$x_3 - x_8 = 5$

$x_1, x_2', x_2'', x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$

e. minimize $6x_1' - 6x_1'' + x_2$

s.t. $-5x_1' + 5x_1'' + 8x_2 + x_3 = 80$

$x_1' - x_1'' + 2x_2 - x_4 = 4$

$x_1', x_1'', x_2, x_3, x_4 \geq 0$

f. minimize $-x_1 - 2x_2 - 4x_3$

s.t.

$3x_1 + 2x_2 - 8x_3 + x_4 = 0$

$-5x_1 - 4x_2 + 6x_3 + x_5 = 0$

$x_1 + x_2 + x_3 - x_6 = 0$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Note:

$|4x_1 + 3x_2 - 7x_3| \leq x_1 + x_2 + x_3$

$\Rightarrow$

$4x_1 + 3x_2 - 7x_3 - x_1 - x_2 - x_3 \leq 0$

$-4x_1 - 3x_2 + 7x_3 - x_1 - x_2 - x_3 \geq 0$

$\Rightarrow$

$3x_1 + 2x_2 - 8x_3 \leq 0$

$-5x_1 - 4x_2 + 6x_3 \leq 0$

$x_1 + x_2 + x_3 \geq 0$

g. minimize $-x_1 - 6x_2 - 12x_3$

s.t.

$-8x_1 - 3x_2 + x_4 - x_5 = 0$

$-x_1 - 6x_2 - x_3 - x_6 = 0$

$x_1 \ldots x_6 \geq 0$

Note:

$-x_1 - x_2 + x_4 \geq \max 7x_1 + 2x_2,\ 5x_2 + x_3 + x_1$

$\Rightarrow$

$-x_1 - x_2 + x_4 \geq 7x_1 + 2x_2$

and

$-x_1 - x_2 + x_4 \geq 5x_2 + x_3 + x_4$

$\Rightarrow$

$-8x_1 - 3x_2 + x_4 \geq 0$

$-x_1 - 6x_2 - x_3 \geq 0$