n) Minimize $-x_1 - x_2 + 2x_3 + x_5$

s.t. $x_1 + 7x_2 + 16x_3 - 4x_4 - x_5 + x_6 = 0$

$-x_1 - 6x_2 + x_3 + 12x_4 - x_7 = 0$

$-x_2 - 3x_4 + 9x_5 + x_8 = 0$

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② Let $x_2 = x_2' - x_0$

$x_3 = x_3' - x_0$

$x_4 = x_4' - x_0$

$x_4', x_3', x_2', x_0 \geq 0$

> Substitute into Problem A to obtain Problem B

Section 3.2

③ a. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ no solution

Not canonical form because the associated basic solution is not feasible (there is no solution)

b. $\begin{bmatrix} 1 & 1 & -3 & 7 \\ -2 & 1 & 5 & 2 \\ 0 & 3 & -1 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -3 & 7 \\ 0 & 3 & -1 & 16 \\ 0 & 3 & -1 & 15 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 1 & -3 & 7 \\ 0 & 3 & -1 & 16 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ no solution

not canonical form bc the associated basic solution is not feasible (no solution)

④ a. $\begin{bmatrix} 2 & 1 & -2 & 17 \\ 1 & 0 & -1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & -2 & 17 \\ 0 & 1 & 0 & 9 \end{bmatrix}$

$x_2 = 9$

$2x_1 - 2x_3 = 17 - 9 = 8$

$x_1 - x_3 = 4$

$\boxed{\begin{matrix} x_2 = 9 \\ x_1 - x_3 = 4 \end{matrix}}$

b. NO

Ex. $\begin{bmatrix}17\\4\end{bmatrix} = 4\begin{bmatrix}2\\1\end{bmatrix} + 9\begin{bmatrix}1\\0\end{bmatrix}$ but $\begin{bmatrix}17\\4\end{bmatrix} \neq c_1\begin{bmatrix}2\\1\end{bmatrix} + c_2\begin{bmatrix}-2\\-1\end{bmatrix}$

⑥ $\begin{bmatrix}1 & 1 & -2 & 3 & 2\\ -2 & 0 & 1 & 0 & 2\end{bmatrix}$

Basic Variable Combinations x_2, x_3 and x_3, x_4

$$x_1 + x_2 - 2x_3 + 3x_4 = 2$$
$$-2x_1 + x_3 = 2$$

x_2, x_3

$x_2 - 2x_3 = 2 \Rightarrow x_2 = 6$ $\quad \begin{bmatrix}0\\6\\2\\0\end{bmatrix}$ feasible ✓

$x_3 = 2$

x_3, x_4

$-2x_3 + 3x_4 = 2 \Rightarrow x_4 = 2$ $\quad \begin{bmatrix}0\\0\\2\\2\end{bmatrix}$ feasible ✓

$x_3 = 2$

✓ (0, 6, 2, 0) and (0, 0, 2, 2)

✓ Bounded Below by 0

Test Both BFS: $\begin{bmatrix}0\\6\\2\\0\end{bmatrix} \Rightarrow z = 12 + 6 = 18$

$\begin{bmatrix}0\\0\\2\\2\end{bmatrix} \Rightarrow z = 6 + 2 = \boxed{8}$