Homework 3
Answer key

Section 3.6

① a. $x_1 - x_2 = 1$
$2x_1 + x_2 - x_3 = 3$

minimize $w = x_4 + x_5$
$x_1 - x_2 + x_4 = 1$
$2x_1 + x_2 - x_3 + x_5 = 3$

Minimize $-3x_1 + x_3 = w - 4$
$x_1 - x_2 + x_4 = 1$
$2x_1 + x_2 - x_3 + x_5 = 3$

	x_1	x_2	x_3	x_4	x_5	
x_4	1	-1	0	1	0	1
x_5	2	1	-1	0	1	3
	-3	0	1	0	0	-4
x_1	1	-1	0	1	0	1
x_5	0	3	-1	-2	1	1
	0	-3	1	3	0	-1
x_1	1	0	-1/3	1/3	1/3	4/3
x_2	0	1	-1/3	-2/3	1/3	1/3
	0	0	0	1	1	0

$\boxed{(4/3, 1/3, 0)}$

b. minimize $w = x_4 + x_5$
s.t. $x_1 + x_2 + x_4 = 1$
$2x_1 + x_2 - x_3 + x_5 = 3$

minimize $-3x_1 - 2x_2 + x_3 = w - 4$
s.t. $x_1 + x_2 + x_4 = 1$
$2x_1 + x_2 - x_3 + x_5 = 3$

	x_1	x_2	x_3	x_4	x_5	
x_4	1	1	0	1	0	1
x_5	2	1	-1	0	1	3
	-3	-2	1	0	0	-4
x_1	1	1	0	1	0	1
x_5	0	-1	-1	-2	1	1
	0	1	1	3	0	-1

$\boxed{\text{no solution}}$

② b. minimize $x_1 - 3x_3$

S.t $x_1 + 2x_2 - x_3 + x_4 = 6$

$x_1 - x_2 + 3x_3 = 3$

minimize $W = x_5 \Rightarrow -x_1 + x_2 - 3x_3 = W - 3$

$x_1 + 2x_2 - x_3 + x_4 = 6$

$x_1 - x_2 + 3x_3 + x_5 = 3$

$x_1 - 3x_3 = z$

	x_1	x_2	x_3	x_4	x_5	
x_4	1	2	-1	1	0	6
x_5	1	-1	3	0	1	3
	1	0	-3	0	0	z
	-1	1	3	0	0	-3
x_4	0	3	-4	1	-1	3
x_1	1	-1	3	0	1	3
	0	1	-6	0	-1	z-3
	0	0	6	0	1	0
x_4	4/3	8/3	0	1	1/3	7
x_3	1/3	-1/3	1	0	1/3	1
	2	-1	0	0	1	z+3
x_2	4/5	1	0	3/5	1/5	21/5
x_3	9/15	0	1	3/15	6/15	36/15
	14/5	0	0	3/5	6/5	z + 36/5

$\boxed{\text{minimum } z = -\frac{36}{3} \ @ \ (0, 21/5, 12/5)}$

f) minimize $8x_1 - 2x_2 - x_3 - 6x_4$

S.t $x_1 + x_2 - x_3 + x_4 = 12$

$-2x_1 + 3x_2 + 2x_4 = 42$

minimize $W = x_5 + x_6 \Rightarrow W - 54 = x_1 - 4x_2 + x_3 - 3x_4$

$x_1 + x_2 - x_3 + x_4 + x_5 = 12$

$-2x_1 + 3x_2 + 2x_4 + x_6 = 42$

$8x_1 - 2x_2 - x_3 - 6x_4 = z$

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	1	1	-1	1	1	0	12
x_6	-2	3	0	2	0	1	42
	8	-2	-1	-6	0	0	z
	1	-4	1	-3	0	0	-54
x_2	1	1	-1	1	1	0	12
x_6	-5	0	3	-1	-3	1	6
	10	0	-3	-4	2	0	$z+24$
	5	0	-3	1	4	0	-6
x_2	-2/3	1	0	2/3	0	1/3	14
x_3	-5/3	0	1	-1/3	-1	1/3	2
	5	0	0	-5	-1	1	$z+30$
x_4	-1	3/2	0	1	0	1/3	21
x_3	-2	1/2	1	0	-1	4/9	9
	0	15/2	0	0	-1	8/3	$z+135$

$-5/3 - 1/3$

$1/3 + \frac{1}{9}$

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$\boxed{\text{minimum } z = -135 \ @ \ (0,0,9,21)}$

(8) $3x_1 + 6x_2 + 6x_3 \geq 36$

$4x_1 + 6x_2 + 3x_3 \geq 20$

$2x_1 + 8x_2 + 4x_3 \geq 30$

minimize $z = 160x_1 + 400x_2 + 300x_3$

minimize $W = x_7 + x_8 + x_9 \Rightarrow$

$3x_1 + 6x_2 + 6x_3 - x_4 + x_7 = 36$

$4x_1 + 6x_2 + 3x_3 - x_5 + x_8 = 20$

$2x_1 + 8x_2 + 4x_3 - x_6 + x_9 = 30$

$160x_1 + 400x_2 + 300x_3 = z$

$w - 86 = -9x_1 - 20x_2 - 13x_3 + x_4 + x_5 + x_6$

$-\frac{320}{3}+\frac{100}{3}$ $\qquad$ $160-\frac{800}{3}$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	
x_7	3	6	6	-1	0	0	1	0	0	36
x_8	4	6	3	0	-1	0	0	1	0	20
x_9	2	8	4	0	0	-1	0	0	1	30
	160	400	300	0	0	0	0	0	0	z
	-9	-20	-13	1	1	1	0	0	0	$w-86$
x_7	-1	0	3	-1	1	0	1	-1	0	16
x_2	$2/3$	1	$1/2$	0	$-1/6$	0	0	$1/6$	0	$10/3$
x_9	$-10/3$	0	0	0	$4/3$	-1	0	$-4/3$	1	$10/3$
	$-320/3$	0	100	0	$200/3$	0	0	$-200/3$	0	$z-4000/3$
	$13/3$	0	-3	1	$-7/3$	1	0	$10/3$	0	$w-50/3$
x_3	$-1/3$	0	1	$-1/3$	$1/3$	0	$1/3$	$-1/3$	0	$16/3$
x_2	$5/6$	1	0	$+1/6$	$-1/3$	0	$-1/6$	$1/3$	0	$2/3$
x_9	$-10/3$	0	0	0	$4/3$	-1	0	$-4/3$	1	$10/3$
	$-\frac{220}{3}$	0	0	$\frac{100}{3}$	$100/3$	0	$1-\frac{100}{3}$	$-\frac{100}{3}$	0	$z-5600/3$
	$10/3$	0	0	0	$-4/3$	1	1	$7/3$	0	$w-2/3$
x_3	$1/2$	0	1	$-1/3$	0	$1/4$	$1/3$	0	$-1/4$	$9/2$
x_2	0	1	0	$1/6$	0	$-1/4$	$-1/6$	0	$1/4$	$3/2$
x_5	$-10/4$	0	0	0	1	$-3/4$	0	-1	$3/4$	$5/2$
	10	0	0	$\frac{100}{3}$	0	25				$z-\frac{5850}{3}$

minimum $z = 1950$ @ $(0, 3/2, 9/2)$

Section 3.9

a.

11 $f(tP + (1-t)Q) = c_1(tp_1 + (1-t)q_1) + c_2(tp_2 + (1-t)q_2) + \ldots + c_n(tp_n + (1-t)q_n)$

$= c_1 t p_1 + c_1(1-t)q_1 + c_2 t p_2 + c_2(1-t)q_2 + \ldots + c_n t p_n + c_n(1-t)q_n$

$= t c_1 p_1 + t c_2 p_2 + \ldots + t c_n p_n + (1-t)c_1 q_1 + \ldots + (1-t)c_n q_n$

$= t(c_1 p_1 + c_2 p_2 + \ldots + c_n p_n) + (1-t)(c_1 q_1 + \ldots + c_n q_n)$

$= t f(P) + (1-t) f(Q)$

b. The optimal value of a convex set in $\mathbb{R}^n$ occurs at a vertex of the convex set in $\mathbb{R}^n$.