Homework 6 Solutions

Section 4.2 # 1, 3
Section 4.3 # 4a, 5ab
Section 4.4 # 2, 3, 6, 10 ab

1) a) minimize $100y_1 + 90y_2 + 500y_3$

$5y_1 - y_2 \geq 20$

$-4y_1 + 12y_2 + y_3 \geq 30$

$y_1, y_2, y_3 \geq 0$

b) $\Rightarrow$ minimize $4x_1 - 3x_2$

s.t. $6x_1 + 11x_2 \geq -30$

$-2x_1 + 7x_2 \geq -50$

$-x_2 \geq -80$

$x_1, x_2 \geq 0$

Dual:

maximize $-30y_1 - 50y_2 - 80y_3$

s.t. $6y_1 - 2y_2 \leq 4$

$11y_1 + 7y_2 - y_3 \leq -3$

$y_1, y_2, y_3 \geq 0$

c) $\Rightarrow$ maximize $-x_1 + 2x_2$

s.t. $5x_1 + x_2 \leq 60$

$-3x_1 + 8x_2 \leq -10$

$x_1 + 7x_2 = 20$

$x_1, x_2 \geq 0$

Dual

minimize $60y_1 - 10y_2 + 20y_3$

s.t. $5y_1 - 3y_2 + y_3 \geq -1$

$y_1 + 8y_2 + 7y_3 \geq 2$

$y_1, y_2 \geq 0$ y_3 unrestricted

d) maximize $30y_1 + 70y_2$

s.t $y_1 + 2y_2 \le 6$

$-3y_1 + 8y_2 \le 12$

$6y_1 - 16y_2 = -18$

y_1, y_2 unrestricted

e) $\Rightarrow$ maximize $x_1 - 7x_2 + 3x_3$

s.t $8x_2 + 5x_3 = 20$

$8x_1 - 3x_3 = 40$

$-x_2 - 4x_3 \le -60$

$x_1, x_2 \ge 0$ x_2 unrestricted

Dual $\to$

minimize $20x_1 + 40x_2 - 60x_3$

$8y_2 \ge 1$

$2y_1 - y_3 = -7$

$5y_1 - 3y_2 - 4y_3 \ge 3$

$y_3 \ge 0$; y_1, y_2 unrestricted

f) $\Rightarrow$ minimize $2y_1 - 3y_2 + 4y_3$

s.t $8y_1 - y_3 = 50$

$-6y_2 - y_3 \ge 60$

$y_3 \ge -15$

$y_1, y_2 \ge 0$ $y_3 \le 0$

Dual $\to$

maximize $50x_1 + 60x_2 - 15x_3$

$4x_1 \le 1$

$2x_2 \ge 1$

$-x_1 - x_2 + x_3 \ge 4$

x_1 unrestricted, $x_2, x_3 \ge 0$

③ Consider: maximizing $6x_1 + x_2 + 4x_3$

Subject to: $3x_1 + 7x_2 + x_3 \le 15$

$x_1 - 2x_2 + 3x_3 \le 20$

$x_1, x_2, x_3 \ge 0$

Dual → minimize $15y_1 + 20y_2$

s.t → $3y_1 + y_2 \ge 6$ ①

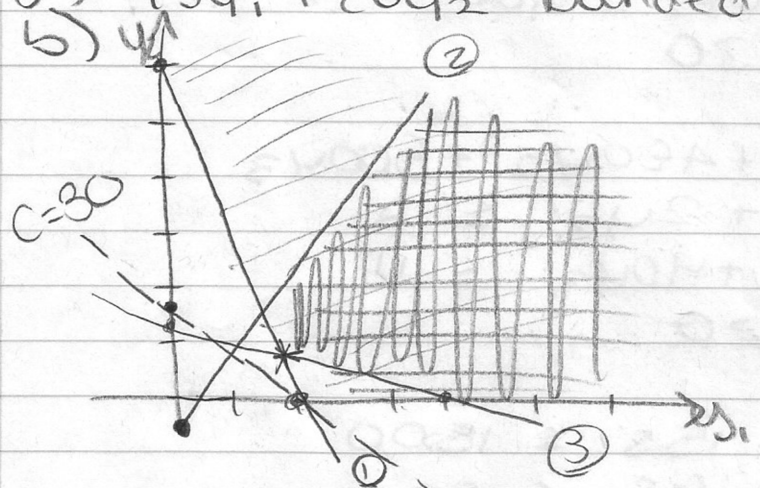
$7y_1 - 2y_2 \ge 1$ ②

$y_1 + 3y_2 \ge 4$ ③

$y_1, y_2 \ge 0$

a) $15y_1 + 20y_2$ bounded below by 0

b)



let $c = 30$

intersection ③ and ①

$y_1 + 3y_2 = 4$

$3y_1 + y_2 = 6$

$-3y_1 - 9y_2 = -12$

$-8y_2 = -6$

$y_2 = 3/4$

$y_1 = 4 - 3(3/4)$

$y_1 = 4 - 9/4$

$y_1 = 7/4$

$15(7/4) + 20(3/4)$ $\boxed{= 41\tfrac{1}{4} \ @ (7/4, 3/4)}$

C_0	x_1	x_2	x_3	x_4	x_5	
x_4	3	7	1	1	0	15
x_5	1	-2	3	0	1	20
	-6	-1	-4	0	0	z
x_1	1	7/3	1/3	1/3	0	5
x_5	0	-13/3	8/3	-1/3	1	15
	0	13	-2	2	0	z +30
x_1	1	23/8	0	3/8	-1/8	25/8
x_3	0	-13/8	1	-1/8	3/8	45/8
	0	35/4	0	7/4	3/4	165/4 + z

$\boxed{z = 41\tfrac{1}{4} \ @ \left(\tfrac{25}{8}, 0, \tfrac{45}{8}\right)}$

d) Bottom two entries in the slack variables column of the original problem are the (y_1, y_2) of the dual problem.

Section 4.3

(4a) ⇒ minimize $2x_1 + 11x_2$

s.t. $-x_1 - 4x_2 \geq -100$

$4x_1 + 20x_2 \geq 480$

$2x_1 + 40x_2 \geq 800$

$x_1, x_2 \geq 0$

Dual:

maximize $-100y_1 + 480y_2 + 800y_3$

$-y_1 + 4y_2 + 2y_3 \leq 2$

$-4y_1 + 20y_2 + 40y_3 \leq 11$

$y_1, y_2, y_3 \geq 0$

(5) a. $2r + 12t + 15s \leq 1500$

$1r + 8t + 6s \leq 920$

maximize $8r + 60t + 45s$

$r, t, s \geq 0$

b. minimize $1500y_1 + 920y_2$

$2y_1 + y_2 \geq 8$

$12y_1 + 8y_2 \geq 60$

$15y_1 + 6y_2 \geq 45$

$y_1, y_2 \geq 0$

Section 4.4

(2) original ⇒ maximize x_1

$$\text{s.t.}\quad \left.\begin{aligned} x_1 - x_2 &\le 1 \\ x_1 - x_2 &\ge 2 \end{aligned}\right\} \boxed{\text{contradiction NO BFS}}$$

$$x_1, x_2 > 0$$

Dual:

minimize $1y_1 - 2y_2$

$$\text{s.t.}\quad \begin{aligned} y_1 - y_2 &\ge 1 \\ -y_1 + y_2 &\ge 0 \end{aligned} \Rightarrow \left.\begin{aligned} y_1 - y_2 &\ge 1 \\ y_1 - y_2 &\le 0 \end{aligned}\right\} \boxed{\text{contradict NO BFS}}$$

$$y_1, y_2 \ge 0$$

(3) a. $54/5,\ 16/5,\ 0,\ 0 \ge 0$

$3(54/5) - 2(16/5) = 26 \le 26$

$54/5 + 6(16/5) = 30 \le 30$

$-4(54/5) + 8(16/5) = -17.6 \le 10$

$\boxed{(54/5, 16/5, 0, 0) \text{ is a basic feasible solution}}$

$4(54/5) + 10(16/5) = \boxed{75.2}$

b. minimize $26y_1 + 30y_2 + 10y_3$

subject to

$$\begin{aligned} 3y_1 + y_2 - 4y_3 &\ge 4 \\ -2y_1 + 6y_2 + 8y_3 &\ge 10 \\ 7y_1 - y_2 - 2y_3 &\ge -3 \\ y_1 + 5y_2 - y_3 &\ge 2 \\ y_1, y_2, y_3 &\ge 0 \end{aligned}$$

$7/10,\ 19/10,\ 0 \ge 0$

$21/10 + 19/10 = 4 \ge 4$

$-14/10 + 114/10 = 10 \ge 10$

$49/10 - 19/10 = 3 \ge -3$

$7/10 + 95/10 = 102/10 \ge 2$

$\boxed{(7/10, 19/10, 0) \text{ is a BFS to the dual}}$

$26(\frac{7}{10}) + 30(\frac{19}{10}) + 0 = \boxed{75.2}$

c) 75.2 is the optimal value of each problem

(6) a. (9, 0, 2, 2) > BFS
(5, 1, 1, 3)

b. at (9, 0, 2, 2)
$z = 157$
at (5, 1, 1, 3)
$z = 116$

c. maximize $32y_1 + 3y_2$
s.t. $4y_1 + 3y_2 \leq 13$
$8y_1 - 2y_2 \leq 15$
$-5y_1 + 6y_2 \leq 12$
$3y_1 - y_2 \leq 8$
y_1 unrestricted $y_2 \geq 0$

d.
BFS: (-1, 0)
(0, 2)

e. @ (-1, 0)
$z = -32$
@ (0, 2)
$z = 6$

f. Since both the original and dual have feasible solutions, the both objective functions will have optimal solutions and they will be equivalent.

(10) a) $\Rightarrow$ minimize $100x_1 + 150x_2$

Subject to

$$2x_1 + x_2 \geq 13$$
$$-6x_1 + 9x_2 \geq -2$$
$$7x_1 - 8x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

Dual:

maximize $13y_1 - 2y_2 + 5y_3$

Subject to

$$2y_1 - 6y_2 + 7y_3 \leq 100$$
$$y_1 + 9y_2 - 8y_3 \leq 150$$
$$y_1, y_2, y_3 \geq 0$$

$(75, 25/3, 0)$

Verify $\Rightarrow$

$75, 25/3, 0 \geq 0$

$150 - 50 + 0 = 100 \leq 100$

$75 + 75 = 150 \leq 150$ BFS $\boxed{\checkmark}$

$13(75) + 50/3 = 2875/3$ = optimal value of original

$\therefore$ optimal $\boxed{\checkmark}$

b) Dual $\Rightarrow$ minimize $9y_1 + 5y_2$

Subject to
$$4y_1 + y_2 \geq 3$$
$$-y_1 + 2y_2 \geq -4$$
$$6y_1 - y_2 \geq 5$$
$y_1 \geq 0$ y_2 unrestricted

$(10/9, -13/9)$

$10/9 \geq 0$

$40/9 - 13/9 = 3 \geq 3$

$-10/9 - 13/9 = -23/9 \geq -4$ BFS $\boxed{\checkmark}$

$60/9 - 13/9 = 47/9 \geq 5$