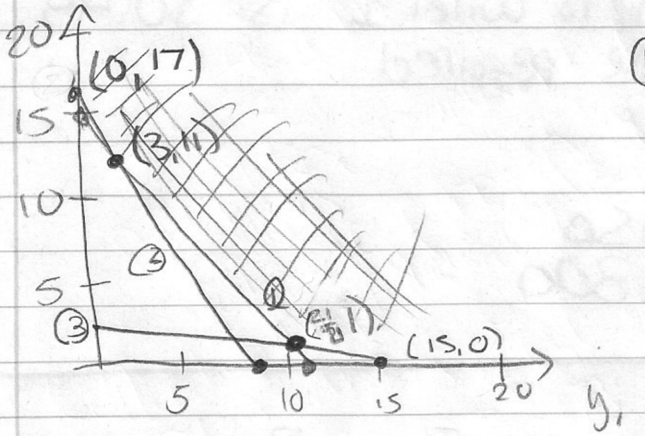
Homework #8
Answer Key

Section 5.1 #7, 13, 17
Section 5.2 #1, 3

(7) x_i = # of product i used

a. $\max z = 45x_1 + 17x_2 + 30x_3$
s.t. $4x_1 + 2x_2 + 2x_3 \le b_1$
$3x_1 + x_2 + 9x_3 \le b_2$
$x_1, x_2 \ge 0$

b. $\min b_1y_1 + b_2y_2$
s.t. $4y_1 + 3y_2 \ge 45$ ①
$2y_1 + y_2 \ge 17$ ②
$2y_1 + 9y_2 \ge 30$ ③
$y_1, y_2 \ge 0$



$\frac{4}{3}$ $\frac{2}{9}$

$\frac{4}{3}$ 2

① and ③
$4y_1 + 3y_2 = 45$
$2y_1 + 9y_2 = 30$
$-15y_2 = -15$
$y_2 = 1$
$2y_1 = 21$
$y_1 = 21/2$

① and ②
$4y_1 + 3y_2 = 45$
$2y_1 + y_2 = 17$
$y_2 = 11$
$y_1 = 3$

if $b_1/b_2 < 2/9$ (15, 0) $z = 15b_1$
if $2/9 < b_1/b_2 < 4/3$ $(\frac{21}{2}, 1)$ $z = 21/2\, b_1 + b_2$
if $4/3 < b_1/b_2 < 2$ (3, 11) $z = 3b_1 + 11b_2$
if $b_1/b_2 > 2$ (0, 17) $z = 17b_2$

(13) a.) we can show
$(300, 0, 0, 300, 0, 350, 400, 150)$
is feasible and gives
$z = 20(300) + 20(300) + 39(350) + 50(400) + 44(150)$
$= 52250$

We can show $(24, 0, 44, 39, 50, 44)$ is also feasible for the dual and gives
$w = -600(24) + 300(44) + 350(39) + 400(50) + 450(44)$
$= 52250$

by The Corollary 4.4.2 these are the optimal solutions.

b) As long as the optimal solution to the original problem is $(300, 0, 0, 300, 0, 350, 400, 150)$ the shadow cost to ship to outlet 1 is \$0.44 for each additional case required.
$0.44 > 0.20$

(17) The "cost" of A is \$150
The "cost" of B is \$300

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(1) a. $B = \begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix}$ $B^{-1} = \frac{1}{4}\begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix}$

$C_B = [0 \ -2]$

b. $b^* = B^{-1} b = \begin{bmatrix} 1/4 & -1/4 \\ 3/4 & 1/4 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$

$$A^{*(1)} = B^{-1}A^{(1)} = \begin{bmatrix} 1/4 & -1/4 \\ 3/4 & 1/4 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A^{*(2)} = B^{-1}A^{(2)} = \begin{bmatrix} 1/4 & -1/4 \\ 3/4 & 1/4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A^{*(3)} = B^{-1}A^{(3)} = \begin{bmatrix} 1/4 & -1/4 \\ 3/4 & 1/4 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$A^{*(4)} = B^{-1}A^{(4)} = \begin{bmatrix} 1/4 & -1/4 \\ 3/4 & 1/4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$$

$$A^{*(5)} = B^{-1}A^{(5)} = \begin{bmatrix} 1/4 & -1/4 \\ 3/4 & 1/4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/4 \\ 1/4 \end{bmatrix}$$

$$A^* = B^{-1}A$$

$$C^* = C - C_B B^{-1} A = [0 \quad -2 \quad -1 \quad 0 \quad 0] - [0 \quad -2]\begin{bmatrix} 1 & 0 & -1 & 1/4 & -1/4 \\ 0 & 1 & -1 & 3/4 & 1/4 \end{bmatrix}$$

$$= [0 \quad 0 \quad -3 \quad 3/2 \quad 1/2]$$

$$Z^* = Z_0 - C_B B^{-1} b = 0 - [0 \; -2]\begin{bmatrix} 1 \\ 6 \end{bmatrix} = 12$$

3 a) $B = \begin{bmatrix} 3 & 2 \\ -6 & 4 \end{bmatrix}$ $\quad B^{-1} = \frac{1}{24}\begin{bmatrix} 4 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1/6 & -1/12 \\ 1/4 & 1/8 \end{bmatrix}$

$C_B = [5 \quad -4]$

b) $Bb^* = b$ $\quad b^* = \begin{bmatrix} b_1^* \\ b_2^* \end{bmatrix}$ $\quad b^* = B^{-1}b = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$b = 5A^{(3)} + 8A^{(4)}$

c) $C^* = C - C_B B^{-1} A = [3 \ 2 \ 5 \ -4 \ 0]$

$$- [5 \ -4] \begin{bmatrix} 1/6 & -1/12 \\ 1/4 & 1/18 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 & 2 & 7 \\ -2 & 3 & -6 & -4 & 1 \end{bmatrix}$$

$$= [3 \ 2 \ 5 \ -4 \ 0] - [-1/6 \ \ -11/12] \begin{bmatrix} 1 & -1 & 3 & 2 & 7 \\ -2 & 3 & -6 & 4 & 1 \end{bmatrix}$$

$$= [3 \ 2 \ 5 \ -4 \ 0] - [5/3 \ \ -31/12 \ \ 5 \ \ -4 \ \ -1/4]$$

$$= [4/3 \ \ 55/12 \ \ 0 \ \ 0 \ \ 1/4]$$