Homework 9
Answer key

§5.3 # 3, 7, 12 §5.4 # 1, 3, 6

(3) a)

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	-2	0	5	1	2	-1	6
x_2	11	1	-18	0	-7	4	4
	3	0	-1	0	2	1	106
x_3	$-2/5$	0	1	$1/5$	$2/5$	$-1/5$	$6/5$
x_2	$\frac{19}{5}$	1	0	$18/5$	$1/5$	$2/5$	$128/5$
	$13/5$	0	0	$1/5$	$12/5$	$4/5$	$107\ 1/5$

maximum @ $(0, 128/5, 6/5, 0)$ $z = 107\frac{1}{5}$

b) $C^* = [-11 \quad -4 \quad -1 \quad -16.5 \quad 0 \quad 0] - [-16.5 \quad -4] \begin{bmatrix} -2 & 0 & 5 & 1 & 2 & -1 \\ 11 & 1 & -18 & 0 & -7 & 4 \end{bmatrix}$

$C^* = [-11 \quad -4 \quad -1 \quad -16.5 \quad 0 \quad 0] - [-11 \quad -4 \quad -10.5 \quad -16.5 \quad -5 \quad .5]$
$= [0 \quad 0 \quad 9.5 \quad 0 \quad 5 \quad -1/2]$

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	-2	0	5	1	2	-1	6
x_2	11	1	-18	0	-7	4	4
	0	0	9.5	0	5	-.5	106
x_4	$3/4$	$1/4$	$1/2$	1	$1/4$	0	7
x_6	$11/4$	$1/4$	$-9/2$	0	$-7/4$	1	1
	$11/8$	$1/8$	$29/4$	0	$33/8$	0	106.5

max @ $(0,0,0,7)$ $z = 106.5$

c) $C^* = [-11 \quad -4 \quad 2 \quad -14 \quad 0 \quad 0] - [-14 \quad -4] \begin{bmatrix} -2 & 0 & 5 & 1 & 2 & -1 \\ 11 & 1 & -18 & 0 & -7 & 4 \end{bmatrix}$

$C^* = [-11 \quad -4 \quad 2 \quad -14 \quad 0 \quad 0] - [-16 \quad -4 \quad 2 \quad -14 \quad 0 \quad -2]$
$C^* = [5 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2]$

no change in optimal variable values
(0, 4, 0, 6)

$\boxed{z = 100}$

(7) $c^{*\prime} = [c_1, c_2, \ldots, c+\lambda, \ldots c_n] - [c_{B1}, c_{B2}, \ldots, c_{Bs}+\lambda, \ldots, c_{Bm}] \begin{bmatrix} a_{11}^* & \cdots & a_{n1}^* \\ \vdots & & \vdots \\ a_{1m}^* & \cdots & a_{nm}^* \end{bmatrix}$

corresponding variable to sth row

$c^* + [0, 0, \ldots, \lambda, 0, 0] - [0, \ldots, \lambda, \ldots, 0]\, A^*$

sth position

$c^* + [0, 0, \ldots \lambda \ldots 0, 0] - \lambda[a_{1s}^*, a_{2s}^*, \ldots, a_{ns}^*]$

$c^* + [-\lambda a_{1s}^*, \ldots, -\lambda a_{ss}^* + \lambda, \ldots, -\lambda a_{ns}^*]$

$[c_1^* - \lambda a_{1s}^*, \ldots, c^* - \lambda a_{ss}^* + \lambda, \ldots, c^* - \lambda a_{ns}^*]$

$\to c_1^* - \lambda a_{1s}^* > 0$

$-\lambda a_{1s}^* > -c_1^*$

if $a_{js}^* > 0$ $\quad \lambda a_{1s}^* < c_1^*$ $\quad$ if $a_{js}^* < 0$

$\lambda < c_1^* / a_{1s}^*$ $\quad \lambda > c_j^* / a_{js}^*$

$\therefore \max\{c_j^*/a_{jr}^* : a_{jr}^* < 0\} \le \lambda \le \min\{c_j^*/a_{jr}^* : a_{jr}^* > 0, j \neq s\}$

(12) a) $c_1^* = c_1 - c_B B^{-1} A^{(1)} =$

$-11 - [-15, 4]\begin{bmatrix} 2 & 1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ \alpha \end{bmatrix} = -5 + \alpha$

$-5 + \alpha > 0$

$\boxed{\alpha > 5}$

b) Yes the analysis would be different because x_2 is a basic variable (x_1 was not)

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1) a) $c_7^* = -9 - [-15 \ \ -4]\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$= -9 - [-15 \ \ -4]\begin{bmatrix} -1 \\ 6 \end{bmatrix} = -9 - (-9) =$
$-9 + 9 = 0$

original solution still optimal $(0,4,0,6;0)$
$z = 106$

b) $c_7^* = -10 - [-15 \ \ -4]\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$-10 + 9 = -1$

$A^{*(7)} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	-2	0	5	1	2	-1	-1	6
x_2	11	1	-18	0	-7	4	6	4
	3	0	2	0	2	1	-1	106
x_4	$-1/6$	$1/6$	2	1	$5/6$	$-1/3$	0	$20/3$
x_7	$11/6$	$1/6$	-3	0	$-7/6$	$2/3$	1	$2/3$
	$29/6$	$1/6$	-1	0	$5/6$	$5/3$	0	$106\ 2/3$
x_3	$-1/12$	$1/12$	1	$1/2$	$5/12$	$-1/6$	0	$20/6 = 10/3$
x_7	$19/12$	$5/12$	0	$3/2$	$1/12$	$1/6$	1	$32/3$
	$19/4$	$3/12$	0	$1/2$	$15/12$	$9/6$	0	110

max @ $(0, 0, 10/3, 0; 32/3)$ $z = 110$

c) $c_7^* = -15 - [-15 \ \ -4]\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} -6 \\ -11 \end{bmatrix}$

$= -15 - [-15 \ \ -4]\begin{bmatrix} -1 \\ -2 \end{bmatrix} = -15 - (23) = -38$

$A^{*(7)} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} -6 \\ -11 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	-2	0	5	1	2	-1	-1	6
x_2	11	1	-18	0	-7	4	-2	4
	3	0	2	0	2	1	-1	106

unbounded solution

d) $c_7^* = -27 - [-15 \ \ -4]\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \end{bmatrix} =$

$-27 - [-2 \ \ -1]\begin{bmatrix} 7 \\ 12 \end{bmatrix} = -27 - (-26) = -1$

$A^{*(7)} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	-2	0	5	1	2	-1	2	6
x_2	11	1	-18	0	-7	4	-1	4
	3	0	2	0	2	1	-1	106
x_7	-1	0	5/2	1/2	1	-1/2	1	3
x_2	10	1	$-15\frac{1}{2}$	1/2	-6	$3\frac{1}{2}$	0	7
	2	0	9/2	1/2	3	1/2	0	109

max $z = 109$ @ $(0,7,0,0;3)$

③ $c_5^* = c_5 - c_B B^{-1} A^{(5)}$

$c_5 = 5$
$c_B = [0 \quad -\tfrac{2}{3}]$

$B = \begin{bmatrix} 1 & -7/5 \\ 0 & 1/5 \end{bmatrix}$

$B^{-1} = \left[\begin{array}{cc|cc} 1 & -7/5 & 1 & 0 \\ 0 & 1/5 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & -7/5 & 1 & 0 \\ 0 & 1 & 0 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & 7 \\ 0 & 1 & 0 & 5 \end{array}\right]$

$B^{-1} = \begin{bmatrix} 1 & 7 \\ 0 & 5 \end{bmatrix}$ $\quad A^{(5)} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$c_5^* = 5 - [0 \quad -2/5] \begin{bmatrix} 1 & 7 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 5 - [0 \quad -2] \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$= 5 + 6 = 11$

original solution holds
$z = 14$ @ $(19, 8, 0, 0, 0)$

⑥ $8(7\tfrac{1}{2}) + 10(0) + 1.5(12) = 60 + 18 = 78$

\$78