MATH381

test #1, 09/08/16

Total 100

Show all work legibly.   **Name:**_____

1. (20) If solutions exist, then find at least one point (x_1, x_2) that solves

$$\begin{cases} 2x_1 + x_2 \leq 2 \\ x_1 + 2x_2 \leq 2 \\ 3x_1 + 3x_2 \geq 4 \end{cases}$$

$(x_1, x_2) =$

2. (40) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$. Consider the system of linear equations $A\mathbf{x} = \mathbf{b}$.

- (20) Select basic variables and free variables.

basic variables are:

free variables are:

- (10) Find general solution for the system $A\mathbf{x} = \mathbf{b}$.

the general solution is:

- (10) Describe the geometric object represented by the solutions.

The geometric object represented by the general solutions is:

3. (20) Let $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{y} \in \mathbf{R}^m$, and A is an $m \times n$ matrix. If $\mathbf{y}^T A \mathbf{x} = 2$ compute $\mathbf{x}^T A^T \mathbf{y}$.

$\mathbf{x}^T A^T \mathbf{y} =$

4. (20) Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a set of three linearly dependent vectors in $\mathbf{R}^n$. Consider two linear combinations of these vectors:

$$\mathbf{a} = t_1\mathbf{u}_1 + t_2\mathbf{u}_2 + t_3\mathbf{u}_3, \text{ and } \mathbf{b} = s_1\mathbf{u}_1 + s_2\mathbf{u}_2 + s_3\mathbf{u}_3.$$

True or False? The vectors $\mathbf{a}$ and $\mathbf{b}$ are linearly dependent.

Mark one and explain.

□ True □ False

5. (20) A library collection contains m distinct books. One has to select n books from the library collection. Compute the number of all different selections.

(Example. Suppose three books are available in the library. They are: "Pinocchio", "War and Peace", and "1984". The different selections of two out of three existing books are

(a) {"Pinocchio", "War and Peace"},

(b) {"Pinocchio", "1984"},

(c) {"War and Peace", "1984"}

Hence in this example there are three different selections of two books.)

the number is: