MATH381

test #3, 12/30/16

Solutions

Total 100

Show all work legibly. **Name**:________________________

1. (30) Solve the LP problem

$$\max \mathbf{c}^T\mathbf{x} = 2x_1 + 4x_2 + 6x_3 + 2x_4 \text{ subject to } \mathbf{x} \geq 0, \quad \begin{array}{rcrcrcrl} x_1 & - & x_2 & + & 2x_3 & + & x_4 & \leq 1 \\ -2x_1 & + & x_2 & + & & + & x_4 & \leq 2 \\ x_1 & + & x_2 & + & x_3 & + & x_4 & \leq 1 \end{array}$$

Solution.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	1	−1	2	1	1	0	0	1
x_6	−2	1	0	1	0	1	0	2
x_7	1	1	1	1	0	0	1	1
	−2	−4	−6	−2	0	0	0	0
x_3	1/2	−1/2	1	1/2	1/2	0	0	1/2
x_6	−2	1	0	1	0	1	0	2
x_7	1/2	3/2	0	1/2	−1/2	0	1	1/2
	1	−7	0	1	3	0	0	3
x_3	2/3	0	1	2/3	1/3	0	1/3	2/3
x_6	−7/2	0	0	2/3	1/3	1	−2/3	5/3
x_2	1/3	1	0	1/3	−1/3	0	2/3	1/3
	10/3	0	0	10/3	2/3	0	14/3	16/3

The optimal solution is: $x_1 = 0$, $x_2 = \frac{1}{3}$, $x_3 = \frac{2}{3}$, $x_4 = 0$.

2. (20) State the dual LP, and provide its solution.

Solution.

$$\min y_1 + 2y_2 + y_3 \text{ subject to } \mathbf{y} \geq 0, \quad \begin{array}{rcrcrl} y_1 & - & 2y_2 & + & y_3 & \geq 2 \\ -y_1 & + & y_2 & + & y_3 & \geq 4 \\ 2y_1 & + & & + & y_3 & \geq 6 \\ y_1 & + & y_2 & + & y_3 & \geq 2 \end{array}$$

The optimal solution is: $y_1 = \frac{2}{3}, y_2 = 0, y_3 = \frac{14}{3}$

3. (20) Add an additional constraint $x_1 + 2x_2 + 2x_3 + x_4 \leq 2$ to LP problem above, and solve it.

Solution. Note that $\left(0, \frac{1}{3}, \frac{2}{3}, 0\right)$ satisfies the new constraint.

4. (30) Find the values $c_1^{'}$ for the cost functional $c_1^{'}x_1 + 4x_2 + 6x_3 + 2x_4$ so that the LP problem in question 1 above has the same optimal solution with $\mathbf{c} = (2, 4, 6, 2)$ and $\mathbf{c}^{'} = (c_1^{'}, 4, 6, 2)$.

Solution. While solving the LP problem we work with the cost $-2x_1 - 4x_2 - 6x_3 - 2x_4$. Since x_1 is not a basic variable in the final tableau any change in $-c_1 = -2$ would generate the exact same change in $c_1^* = \frac{10}{3}$. Hence adding λ to -2 would add λ to $\frac{10}{3}$, and we need $\frac{10}{3} + \lambda \geq 0$, i.e. $\lambda \geq -\frac{10}{3}$. Hence $-c_1 + \lambda \geq -2 - \frac{10}{3}$, and

$$c_1' = c_1 - \lambda \leq \frac{16}{3}.$$