

MATH221

Midterm #1, 02/25/16

Total 100

Solutions

Show all work legibly.

Name: _____

1. (25) Solve the system

$$\begin{array}{rcl} x_1 + 5x_2 + 8x_3 & = & 0 \\ 2x_1 & & -4x_3 = -10 \\ & & x_2 + 3x_3 = 2 \end{array}$$

Solution.

$$\begin{aligned} \begin{bmatrix} 1 & 5 & 8 & 0 \\ 2 & 0 & -4 & -10 \\ 0 & 1 & 3 & 2 \end{bmatrix} &\rightarrow \begin{bmatrix} 1 & 5 & 8 & 0 \\ 0 & -10 & -20 & -10 \\ 0 & 1 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 5 & 8 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 5 & 8 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \\ &\begin{bmatrix} 1 & 5 & 0 & -8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{aligned}$$

$$x_1 = -3, x_2 = -1, x_3 = 1$$

2. (25) Determine values of h for which the system

$$-4x_1 + 12x_2 = 0, hx_1 - 6x_2 = -3$$

- (15) Has no solutions.

Solution.

$$\begin{bmatrix} -4 & 12 & 0 \\ h & -6 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 \\ h & -6 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 3h-6 & -3 \end{bmatrix}$$

$$h = 2$$

- (10) Has only one solution

Solution. See above.

$$h \neq 2$$

3. (25) Let $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

(a) (10) Find A^{-1} if exists.

$$A^{-1} = A.$$

(b) (15) If B is a 2×3 matrix so that $AB = C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. Find B .

Solution. $AB = C$, hence $B = A^{-1}C = \begin{bmatrix} -4 & -5 & -6 \\ -1 & -2 & -3 \end{bmatrix}$.

$$B =$$

4. (25) True or False? The vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

are linearly dependent.

Solution. $\mathbf{v}_1 - \mathbf{v}_2 + \frac{1}{3}\mathbf{v}_3 = 0$

Mark one and explain.

True False