MATH410

Take Home Final Examination 05/10/16

Submission Deadline: Monday, May 16, 2016

Total 200

Show all work legibly. **Name**:______________________

1. (20) Let $z_1 \neq z_2$. Describe the set of complex numbers $\Omega = \{w \ : \ |z_1 - w| = |z_2 - w|\}$.

$\Omega =$

2. (20) Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$, $z_3 = x_3 + iy_3$, be three distinct complex numbers that do not belong to the same line. Describe the set of complex numbers

$$\Omega = \{w \ : \ |z_1 - w| = |z_2 - w| = |z_3 - w|\}.$$

$\Omega =$

3. (20) Let $z_0 = 2 + 4i$, and $\gamma = \{z \ : \ |z - z_0| = \sqrt{5}\}$ (i.e. γ is a circle of radius $\sqrt{5}$ centered at z_0). Compute the shortest distance d between $w = 1 + i$ and the set $\Gamma = \dfrac{1}{\gamma} = \left\{w \ : \ w = \dfrac{1}{z}, \ z \in \gamma\right\}$.

$d =$

4. (20) For what values of z the absolute value of the n-th term $\frac{z^n}{n!}$ of

$$1 + \frac{z}{1!} + \frac{z^2}{2!} + \ldots + \frac{z^n}{n!} + \ldots$$

larger than the absolute value of any other term? $(n = 0, 1, \ldots)$

The values of z are:

5. (20) Let $f(z)$ be an entire function such that $|f(z)| > 1$ for each z. True or False? f is a constant.

Mark one and explain.

□ True □ False

6. (20) Let $f(z)$ be an entite function such that $|f(z)| \leq |z|^n$ if $|z| > 1$. True or False? $f(z)$ is a polynomial of degree n or less.

Mark one and explain.

□ True □ False

7. (20) Compute $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x^2+1}\, dx.$

$$\int_{-\infty}^{\infty} \frac{\sin x}{x^2+1}\, dx =$$

8. (20) Compute $\int_{-\infty}^{\infty} \frac{e^{-ix}}{x^2+1}\, dx$ (Hint: use residue.)

$$\int_{-\infty}^{\infty} \frac{e^{-ix}}{x^2+1}\, dx =$$

9. (20) Suppose f and g are analytic on and inside closed simple curve γ and $f(z) = g(z)$ for each $z \in \gamma$. True or False? $f(w) = g(w)$ for each w inside γ.

Mark one and explain.

□ True □ False

10. (20) Suppose $\gamma = \{z : |z| = 1\}$ is a unit circle centered at the origin, and $g(z)$ is continious on γ. Define $G(z) = \int_\gamma \frac{g(w)}{w - z}\, dw$ for each $z \notin \gamma$. Show that $G(z)$ is differentiable, and compute $G'(z)$. (Note that Cauchy's formula requires the function to be analytic on a domain bounded by a closed curve. The function $g(z)$ above is not necessarily analytic.)

$G'(z) =$