1. General

   1.1. **(Maxim Marten, Noel Diaz, and Cassia Connolly, 10/10, September 27, 2017)**

   Max and Dani are cycling between Baltimore, MD and Washington, DC. The Max is cycling from from Baltimore, MD to Washington, DC. His speed is 12 m/h. Dani is cycling from Washington, DC to Baltimore, MD. His speed is 8 m/h. The moment Max starts to move a small fly takes off his nose and flies toward Dani. Once the fly reaches Dani it turns back and flies toward Max. The fly is in the air until the cyclists meet. The fly's speed is 15 m/h. How much distance did the fly cover until the cyclists met (according to Google Maps the distance between Baltimore and Washington is 40 miles).

   1.2. (**Imaan Kibria, Gabrielle Millard, and Cade Simon, 1/10, October 31, 2017**)

   A hunter walks a mile due south, turns and walks a mile due east, turns again and walks a mile due north, only to find herself back where she started. The hunter draws a bead on a bear and shoots it dead. What color is the bear? Why?

   1.3. Three lawyers rent a hotel room for the night. When they get to the hotel they pay the $30 fee, then go up to their room. Soon the bellhop brings up their bags and gives the lawyers back $5 because the hotel was having a special discount that weekend. So the three lawyers decide to each keep one of the $5 dollars and to give the bellhop a $2 tip. However, when they sat down to tally up their expenses for the weekend the could not explain the following details:

Each one of them had originally paid \$10 (towards the initial \$30), then each got back \$1 which meant that they each paid \$9. Then they gave the bellhop a \$2 tip. HOWEVER, $3 \times \$9 + \$2 = \$29$.

The guys couldn't figure out what happened to the other dollar. After all, the three paid out \$30 but could only account for \$29. Can you determine what happened?

1.4. What digit is the most frequent between the numbers 1 and 1,000 (inclusive)? To solve this problem you don't want to manually do all of the math but rather try to figure out a pattern. What digit is the least frequent between the numbers 1 and 1,000?

1.5. A merchant can place 8 large boxes or 10 small boxes into a carton for shipping. In one shipment, he sent a total of 96 boxes. If there are more large boxes than small boxes, how many cartons did he ship?

2. Mathematical Induction

2.1. **(Maxim Marten, Noel Diaz, and Cassia Connolly, 10/10, October 4, 2017)**
Show that $1^2 + 2^2 + \ldots + n^2 = \dfrac{(2n+1)(n+1)n}{6}$.

2.2. **(Maxim Marten, Noel Diaz, and Cassia Connolly, 10/10, October 4, 2017)**
Show that $1^3 + 2^3 + \ldots + n^3 = \left(\dfrac{(n+1)n}{2}\right)^2$.

2.3. **(Maxim Marten, Noel Diaz, and Cassia Connolly, 10/10, October 4, 2017)**
Note that

$$\begin{array}{lllllll} 1 & = & 1^2 & & & & \\ 1 & + & 3 & = & 2^2 & & \\ 1 & + & 3 & + & 5 & = & 3^2 \end{array}$$

Show that $1+3+5+\ldots+2n-1=n^2$.

2.4. Consider the sum $1+x+x^2+\ldots+x^n$. If $x=1$, then

$$1+x+x^2+\ldots+x^n = 1+1^2+\ldots+1^n = \underbrace{1+1+\ldots+1}_{n+1 \text{ times}} = n+1.$$

Show that if $x \neq 1$, then $1+x+x^2+\ldots+x^n = \dfrac{x^{n+1}-1}{x-1}$.

2.5. Show that $1\times2+2\times3+\ldots+(n-3)\times(n-2) = \dfrac{(n-3)(n-2)(n-1)}{3}$.

2.6. Show that

$$(a+b)^n = \frac{n!}{(n-0)!0!}a^nb^0+\frac{n!}{(n-1)!1!}a^{n-1}b^1+\ldots+\frac{n!}{(n-n)!n!}a^{n-n}b^n.$$

Here $0! = 1$, and $n! = 1\cdot2\ldots(n-1)\cdot n$ for $n \geq 1$.

2.7. Show that $1\cdot2\cdot3+2\cdot3\cdot4+k(k+1)(k+2) = \dfrac{k(k+1)(k+2)(k+4)}{4}$.

2.8. Show that $\dfrac{1}{\sqrt{1}}+\dfrac{1}{\sqrt{2}}+\ldots+\dfrac{1}{\sqrt{k}} > \sqrt{k}$.

3. Division

3.1. **(Maxim Marten, Noel Diaz, and Cassia Connolly, 8/10, September 27, 2017)**

if n is even, then n^2 is even. if n^2 is even, then n is even.

3.2. $\sqrt{2}$ is not a ratio of two integers.

3.3. if sum of the digits of an integer is divisible by 3, the integer is divisible by 3.

3.4. (**Imaan Kibria, Gabrielle Millard, and Cade Simon, 10/10, October 31, 2017**)

Let ab be a two digit integer. The integer $ab-ba$ is divisible by 3.

4. Inequalities

4.1. Show that for all non negative numbers x and y one has

$$\sqrt{x}+\sqrt{y}\le\sqrt{2}\sqrt{x+y}.$$

4.2. Show that for all non negative numbers x, y, and z one has

$$\sqrt{x}+\sqrt{y}+\sqrt{z}\le\sqrt{3}\sqrt{x+y+z}.$$

4.3. Can you generalize Problem 4.2 to the case of n non negative numbers?

4.4. Show that for all positive numbers x, y, and z one has

$$\sqrt{\frac{x+y}{x+y+z}}+\sqrt{\frac{y+z}{x+y+z}}+\sqrt{\frac{z+x}{x+y+z}}\le\sqrt{6}.$$

4.5. If $x>0$, then $x+\dfrac{1}{x}\ge 2$.

4.6. Show that $\dfrac{1}{\sqrt{k+1}}>\sqrt{k+1}+\sqrt{k}$.

5. Geometry

5.1. A 20 ft tall tree is 50 ft away from a 25 ft tall tree. A little bird is sitting on the top of the shorter tree. The bird need to fly to the top of the taller tree, but can not do it without landing. Determine a landing point so that the flight trajectory is shortest possible.

5.2. Given n points in the plane determine the maximal number of straight lines crossing pairs of these points.

6. Set Theory

6.1. Let $A=\{x\ :\ 0\le x\le 1\}$, and $B=\{x\ :\ 0<x\le 1\}$. Show that A is equivalent to B.

6.2. Let $A = \{x : 0 < x < 1\}$, and $B = \{x : -\infty < x < \infty\}$. Show that A is equivalent to B.

6.3. Let $A = \{x : 0 \le x \le 1\}$, and $B = \{(x, y) : 0 \le x, y \le 1\}$. Show that A is equivalent to B.