Name:

MATH120
quiz #1, 09/07/17
Total 120
Solutions

Show all work legibly.

1. (20) Solve $x^3 + 2x^2 + x = 0$.

   Solution.
   $$x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x+1)^2 = 0.$$
   $x = 0$ or $x = -1$.

2. (20) How many positive integer solutions (x, y) does the equation
   $$x + y = 1000$$
   have?

   Solution. The solutions are: $x = 1, \ldots, 999$, $y = 1000 - x$. Hence there are all together 999 integer positive solutions.

3. (20) Find x so that $x + 1^x = 3$.

   Solution. Since $1^x = 1$, one has $x = 2$.

4. (20) True or False? $5^{\frac{1}{5}} < 4^{\frac{1}{4}}$.

   Solution.
   $$\left(5^{\frac{1}{5}}\right)^{10} = 5^2 < 2^5 = \left(2^{\frac{1}{2}}\right)^{10} = \left(4^{\frac{1}{4}}\right)^{10}.$$
   Mark one and explain.
   ▫ True ▫ False

5. (20) Let x and y be non negative numbers. True or False? $\sqrt{xy} \leq \dfrac{x+y}{2}$

   Solution.
   $$\begin{aligned} 0 &\leq (x-y)^2 = x^2 - 2xy + y^2 \\ 4xy &\leq x^2 + 2xy + y^2 \\ xy &\leq \frac{x^2 + 2xy + y^2}{4} = \frac{(x+y)^2}{4} \\ \sqrt{xy} &\leq \frac{x+y}{2} \end{aligned}$$
   Mark one and explain.
   ▫ True ▫ False

6. (20) Let $a - b = -1$. Compute $a^2 - 2ab + b^2$.

   Solution. $a^2 - 2ab + b^2 = (a-b)^2 = 1$.