MATH 225, FALL 2017 - HOMEWORK #5

Due Thursday, October 19

Section 4.4, page 182: 9, 11, 20, 23, 26
Section 4.6, page 193: 1, 7, 12, 17

Section 4.4

Problem 9: For the equation$y'' + 2y' - y = 10$, we look for a particular solution of the form $y_p = A$. Then, $y_p' = y_p'' = 0$. Substituting that back into the equation, we arrive at $-A = 10$, so $A = -10$. Therefore, a particular solution is $y_p(x) = -10$.

Problem 11: For the equation $y'' + y = 2^x$, we look for a particular solution of the form $y_p = A2^x$. Then, $y_p' = A2^x \ln(2)$ and $y_p'' = A2^x(\ln 2)^2$. Substituting that back into the equation, we arrive at $A2^x(\ln 2)^2 + A2^x = 2^x$, so $2^x(A(\ln 2)^2 + A) = 2^x$. Therefore, $A = \frac{1}{(\ln 2)^2+1}$. Therefore, a particular solution is $y_p(x) = \frac{2^x}{(\ln 2)^2+1}$.

Problem 20: For the equation $y'' + 4y = 16t \sin 2t$, we first look at the solution to the homogeneous equation: $y'' + 4y = 0$, which is $y_h(t) = c_1 \cos(2t) + c_2 \sin(2t)$. Since 2 is a root in the auxillary equation, we look for a solution of the form $y_p = (At^2 + B^t)\cos(2t) + (Ct^2 + Dt)\sin(2t)$. Calculate y_p' and y_p'' and then substitute these into the original equation and simplify to get a system of equations in terms of the coefficients. Solving that system yields the particular solution $y_p = -4t\cos(2t)$.

Problem 23: For the equation $y'' - 7y' = \theta^2$, we first check the solution to the homogeneous equation: $y'' - 7y' = 0$, which is $y(\theta) = c_1 + c_2 e^{7\theta}$. Since 0 is a root in the homogeneous equation, we look for a particular solution of the form $y_p = A\theta^3 + B\theta^2 + C\theta + D$. Therefore, $y_p' = 3A\theta^2 + 2B\theta + C$ and $y_p'' = 6A\theta + 2B$. Substituting this into the original equation, we get $6A\theta + 2B - 21A\theta^2 - 14B\theta - 7C = \theta^2$. Therefore, we get the system of equations $\begin{array}{c} 6A - 14B = 0 \\ 2B - 7C = 0 \\ -21A = 1 \end{array}$. Therefore, $A = -\frac{1}{21}$, $B = -\frac{1}{49}$, and $C = -\frac{2}{343}$ so $y_p = -\frac{1}{21}\theta^3 - \frac{1}{49}\theta^2 - \frac{2}{343}\theta$.

Problem 26: For the equation $y'' + 2y' + 2y = 4te^{-t}\cos t$, we first check the solution to the homogeneous equation: $y'' + 2y' + 2y = 0$, which is $y(t) = c_1 e^{-t}\sin(t) + c_2 e^{-t}\cos(t)$. Since we have $e^{-t}\cos(t)$ in the homogeneous solution, we look for a particular solution of the form $y_p = (At^2 + Bt)e^{-t}\cos(t) + (Ct^2 + Dt)e^{-t}\sin(t)$. Then, we get $y_p' = (C - A)t^2 e^{-t}\cos t + (-A - C)t^2 e^{-t}\sin t + (2A - B + D)te^{-t}\cos t + (-A + 2C - D)te^{-t}\sin t + Ae^{-t}\cos t + De^{-t}\sin t$ and $y_p'' = -2Ct^2 e^{-t}\cos t + 2At^2 e^{-t}\sin t + (4C - 4A - 2D)te^{-t}\cos t + (-4A + 2B - 4C)te^{-t}\sin t + (2A - 2B + 2D)e^{-t}\cos t + (-2A + 2C - 2D)e^{-t}\sin t$. Substituting into the original equation yields$4Cte^{-t}\cos(t) - 4Ate^{-t}\sin t + (2A + D)e^{-t}\cos t + (-2B + 2C)e^{-t}\sin t = 4te^{-t}\cos t$. The systems of equations is $\begin{array}{c} 4C = 4 \\ -4A = 0 \\ 2A + D = 0 \\ -2B + 2C = 0 \end{array}$, so $B = C = 1$ and $A = D = 0$. Therefore, the particular solution is $y_p = te^{-t}\cos t + t^2 e^{-t}\sin t$.

Section 4.6

Problem 1: First, we must solve the homogeneous equation $y'' + y = 0$, which is $y(x) = c_1 \cos x + c_2 \sin x$. Therefore, we look for a particular solution $y_p = v(x)\cos x + u(x)\sin x$. Using the formula for variation of parameters, we find $v(x) = \int \frac{-\sec x \sin x}{\cos^2 x + \sin^2 x}dx = \int -\tan x \, dx = \ln(\cos x)$ and $u(x) = \int \frac{\sec x \cos x}{\cos^2 x + \sin^2 x}dx = \int dx = x$. therefore, the particular solution is $y_p = \ln(\cos x)\cos x + x\sin x$. Therefore, the general solution is the sum of the homogeneous solution and particular solution: $y(x) = c_1 \cos x + c_2 \sin x + \ln(\cos x)\cos x + x\sin x$.

Problem 7: First, we must solve the homogeneous equation $y'' + 4y' + 4y = 0$, which is $y(t) = c_1 e^{-2t} + c_2 te^{-2t}$. Therefore, we look for a particular solution $y_p = v(t)e^{-2t} + u(t)te^{-2t}$. Using the formula for variation of parameters, we find $v(t) = \int \frac{(-e^{-2t}\ln t)te^{-2t}}{e^{-2t}(e^{-2t}-2te^{-2t})+e^{-2t}2te^{-2t}}\,dt = \int -t\ln t\,dt = \frac{1}{4}t^2(1-2\ln t)$ and $u(t) = \int \frac{(e^{-2t}\ln t)e^{-2t}}{e^{-2t}(e^{-2t}-2te^{-2t})+e^{-2t}2te^{-2t}}\,dt = \int \ln t\,dt = t\ln t - t$. therefore, the particular solution is $y_p = e^{-2t}(\frac{1}{4}t^2(1-2\ln t)) + te^{-2t}(t\ln t - t)$. Therefore, the general solution is the sum of the homogeneous solution and particular solution: $y(x) = c_1 e^{-2t} + c_2 te^{-2t} + e^{-2t}(\frac{1}{4}t^2(1-2\ln t)) + te^{-2t}(t\ln t - t)$.

Problem 12: First, find the solution to the homogeneous equation: $y'' + y = 0$. It is $y(t) = c_1\cos t + c_2\sin t$. Consider $y'' + y = \tan t$, $y'' + y = e^{3t} - 1$. A particular solution to the first equation is found in Example 1 of this section using the variation of parameters. It is $y_p = -\cos t\ln(\sec t + \tan t)$. A particular solution to the second equation can be found using the method of undetermined coefficients. The solution is $y_p = (1/10)e^{3t} - 1$. By the Superposition Principle, a particular solution to the original equation is $y_p = -\cos t\ln(\sec t + \tan t) + (1/10)e^{3t} - 1$. Therefore, the general solution is $y(t) = c_1\cos t + c_2\sin t + -\cos t\ln(\sec t + \tan t) + (1/10)e^{3t} - 1$.

Problem 17: In this problem, you must use the variation of paramters. The solution is $y(x) = c_1\cos 2t + c_2\sin 2t - e^t/5 - (1/2)(\cos 2t)\ln(\sec 2t + \tan 2t)$.