MATH225

quiz #1, 09/28/17

Total 120

Solutions

Show all work legibly. **Name**:______________________

1. (20) Consider $y' - xy = x$.

   (a) (5) Identify $y' - xy = x$.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

   (b) (15) Solve $y' - xy = x$.

   The solution is:

   Solution.

$$\begin{aligned} y' - xy &= x \\ y' &= x(1+y) \\ \frac{y'}{1+y} &= x \\ y(x) &= ce^{x^2/2} - 1 \end{aligned}$$

2. (20) Consider the differential equation $(4+t^2)y'(t) + 2ty(t) = 4t$.

   (a) (5) Identify the equation.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

   (b) (15) Solve the equation.

   The solution is:

   Solution. Since

$$\frac{d}{dt}[(4+t^2)y] = (4+t^2)y'(t) + 2ty(t) = 4t$$

   one has $(4+t^2)y = 2t^2 + c$ and $y(t) = \frac{2t^2}{4+t^2} + \frac{c}{4+t^2}$.

3. (20) Consider $y'(x) = \frac{x^2}{1-y^2}$.

   (a) (5) Identify the equation.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

(b) (15) Solve the equation.

The solution is:

Solution. We can rewrite the equation as $(1-y^2)y'(x) = x^2$, and $y - \dfrac{y^3}{3} - \dfrac{x^3}{3} = c$.

4. (20) Consider $y' + \dfrac{4}{x}y = x^3y^3$.

(a) (5) Identify the equation.

▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

(b) (15) Solve the initial value problem

$$y' + \frac{4}{x}y = x^3y^3,\ y(1) = 1,\ x > 0.$$

The solution is:

Solution. The substitution $u = y^{-2}$ leads to the linear equation $u' - \dfrac{8}{x}u = -2x^3$. The solution of this equations is $u(x) = \frac{1}{2}x^4 + cx^8$, and $y(x) = \left(\dfrac{1}{2}x^4 + cx^8\right)^{-\frac{1}{2}}$. The initial condition yields $c = \dfrac{1}{2}$.

5. (20) Solve the initial value problem $y' = \sqrt{2x+y} - 2$, $y(0) = 1$.

(a) (5) Identify the equation.

▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

(b) (15) Solve the equation.

The solution is:

Solution. Let $u = 2x + y$. Then $u' = 2 + y'$, hence

$$u' = 2 + y' = 2 + \sqrt{u} - 2,\ \text{and}\ u' = \sqrt{u}.$$

The solution for the separable equation is $2u^{\frac{1}{2}} = x + c$, and $u = \left(\dfrac{x+c}{2}\right)^2$, and $y = \left(\dfrac{x+c}{2}\right)^2 - 2x$. Substitution $y(0) = 1$ leads to $c = \pm 2$.

6. (20) Identify and solve $(e^{4y} + 2x)dy - dx = 0$.

(a) (5) Identify the equation.

□ separable □ linear □ exact □ homogeneous □ Bernoulli □ none of the above

(b) (15) Solve the equation.

The solution is:

Solution. $\dfrac{dx}{dy} = e^{4y} + 2x$, and $\dfrac{dx}{dy} - 2x = e^4 y$. The general solution of this linear equation is $x = \dfrac{1}{2}e^{4y} + ce^{2y}$.