MATH225
quiz #2, 10/19/17
Total 120
Solutions

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Show all work legibly. **Name**:______________________

1. (20) Find the general solution $y_g(x)$ for $y'' - 2y' + y = 0$.

   The solution is:

   Solution. $0 = r^2 - 2r + 1 = (r-1)^2$, hence $y_g(x) = c_1 e^x + c_2 x e^x$.

2. (50) Consider $y'' + 4y = 3\csc\ t$
   (Reminder $\csc t = \dfrac{1}{\sin t}$).

   (a) (5) Find the characteristic equation for $y'' + 4y = 0$ and factor it.

   The solution is: $0 = r^2 + 4 = (r - 2i)(r + 2i)$.

   (b) (15) Solve $y'' + 4y = 0$.

   The solution is:
   Solution. $y(t) = c_1 \cos(2t) + c_2 \sin(2t)$.

   (c) (20) Find a particular solution $y_p(t)$ for $y'' + 4y = 3\csc\ t$.

   The solution is:
   Solution. A particular solution $y_p(t)$ to the equation $y'' + 4y = 3\csc\ t$ is

   $$y_p(t) = u_1(t)\cos(2t) + u_2(t)\sin(2t),$$

   where $u_1'(t) = -3\cos\ t$, and $u_2'(t) = -3\sin(t) + \dfrac{3}{2}\csc t$. Integration of $u_1'(t)$ and $u_2'(t)$ leads to

   $$u_1(t) = -3\sin t \text{ and } u_2(t) = \frac{3}{2}\ln|\csc t - \cot t| + 3\cos t.$$

   . Finally $y_p(t) = -3\sin t\cos(2t) + \left(\dfrac{3}{2}\ln|\csc t - \cot t| + 3\cos t\right)\sin(2t)$.

   The solution is:

(d) (10) Find the general solution $y_g(t)$ for $y'' + 4y = 3\csc\ t$.

Solution.

$$y_g(t) = -3\sin t\cos(2t) + \left(\frac{3}{2}\ln|\csc t - \cot t| + 3\cos t\right)\sin(2t) + c_1\cos(2t) + c_2\sin(2t).$$

The solution is:

3. (50) Consider the differential equation $y'' - 3y' - 4y = 2e^{-t}$.

(a) (5) Find the characteristic equation for $y'' - 3y' - 4y = 0$ and factor it.
The characteristic equation is: $0 = r^2 - 3r - 4 = (r-4)(r+1)$.

(b) (15) Solve $y'' - 3y' - 4y = 0$.

The solution is:
Solution. $y(t) = c_1e^{4t} + c_2e^{-t}$.

(c) (20) Find a particular solution $y_p(t)$ for $y'' - 3y' - 4y = 2e^{-t}$.

The solution is:
Solution. A particular solution is $y_p(t) = Ate^{-t}$, where $A = -\frac{2}{5}$.

(d) (10) Find the general solution $y_g(t)$ for $y'' - 3y' - 4y = 2e^{-t}$.
The solution is: $y_g(t) = c_1e^{4t} + c_2e^{-t} - \frac{2}{5}te^{-t}$.