MATH225
quiz #1, 02/28/17
Total 100
Solutions

Show all work legibly. **Name**:______________________

1. (20) Consider $y' + 4xy = x$.

   (a) (5) Identify $y' + 4xy = x$.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

   (b) (15) Solve $y' + 4xy = x$.

   The solution is:

   Solution.

$$\begin{aligned} y' + 4xy &= x \\ y' &= x(1-4y) \\ \frac{y'}{1-4y} &= x \\ y(x) &= ce^{-2x^2} + \frac{1}{4} \end{aligned}$$

2. (20) Consider $(y^2 - 2x) + (2xy + 1)y'(x) = 0$.

   (a) (5) Identify the equation.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

   (b) (15) Solve the equation.

   The solution is:

   Solution. Since

$$\frac{\partial(y^2 - 2x)}{\partial y} = 2y = \frac{\partial(2xy+1)}{\partial x}$$

   this is an exact equation. We selets $(x_0, y_0) = (0, 0)$ and obtain

$$\begin{aligned} F(x,y) &= F(0,0) + \int_0^x [-2t]\ dt + \int_0^y [2xs + 1]\ ds \\ &= F(0,0) - t^2 \,|_0^x + \left[xs^2 + s\right] |_0^y \\ &= F(0,0) - x^2 + xy^2 + y. \end{aligned}$$

   The solution is $F(x, y) = -x^2 + xy^2 + y = c$.

3. (20) Consider $y'(x) = \dfrac{y^2 - x^2}{xy}$ for $x > 0$.

   (a) (5) Identify the equation.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

   (b) (15) Solve the equation.

   The solution is:

   Solution. If $u(x) = \dfrac{y(x)}{x}$, then $xu = y$, and

$$u + xu' = y' = u - \frac{1}{u}, \text{ and } uu' = -\frac{1}{x}, \text{ and } u^2(x) = -2\left(\ln x + c\right).$$

   Finally $y^2(x) = -2x^2 (\ln x + c)$.

4. (20) Consider $y' - y = y^3$.

   (a) (5) Identify the equation.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

   (b) (15) Solve the equation.

   The solution is:

   Solution. Substitute $u = y^{1-3} = \dfrac{1}{y^2}$ (at some point you will have to check the possible solution $y(x) = 0$). After the substitution the equation becomes

$$-\frac{1}{2}u'(x) - u(x) = 1, \text{ and } u'(x) + 2u = -2.$$

   The later is a linear equation. With $\mu(x) = \exp\left(\int^x 2\, ds\right) = e^{2x}$, one has

$$\frac{1}{y^2} = u = e^{-2x}\left[\int^x e^{2s}(-2)\, ds + c\right] = -1 + ce^{-2x}.$$

5. (20) Solve the initial value problem $y' = 2 - \sqrt{2x - y + 3}$, $y(0) = 2$.

   (a) (5) Identify the equation.

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ none of the above

(b) (15) Solve the equation.

The solution is:

Solution. Let $u = 2x - y + 3$. Then $y = 2x - u + 3$, and $y' = 2 - u'$, hence

$$y' = 2 - u' = 2 - \sqrt{u}, \text{ and } u' = \sqrt{u}.$$

The solution for the separable equation is $2u^{\frac{1}{2}} = x + c$, and $u = \left(\frac{x+c}{2}\right)^2$, and $y = 2x - \left(\frac{x+c}{2}\right)^2 + 3$. Substitution $y(0) = 2$ leads to $y = 2x - \left(\frac{x+2}{2}\right)^2 + 3$.