MATH225
quiz #2, 03/30/17
Total 100
Solutions

Show all work legibly. **Name**:______________________

1. (10) Find the general solution $y_g(x)$ of $y_g(x)$ of $y'' - y' - 6y = 0$.

   The general solution $y_g(x) =$
   Solution. $r^2 - r - 6 = 0$, hence $y_g(x) = c_1 e^{-2x} + c_2 e^{3x}$.

2. (10) Find the general solution $y_g(x)$ of $y_g(x)$ of $y'' - 2y' + y = 0$.

   The general solution $y_g(x) =$
   Solution. $0 = r^2 - 2r + 1 = (r-1)^2$, $y_g(x) = c_1 e^x + c_2 x e^x$.

3. (10) Find the general solution $y_g(x)$ of $y'' + y' + y = 0$.

   The general solution $y_g(x) =$

   Solution. $0 = r^2 + r + 1$, $r = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. Hence

$$y_g(x) = c_1 e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{-\frac{1}{2}x} \sin\left(\frac{\sqrt{3}}{2}x\right).$$

4. (20) Given that $y_1(x) = \frac{1}{x}$ is one solution of $2x^2y'' + 3xy' - y = 0$, $x > 0$ find a second linearly independent solution $y_2(x)$.

   The second linearly independent solution is: $y_2(x) =$

   Solution. Set $y_2(x) = \frac{c(x)}{x}$. Substituting for y, y', and y'' into the equation we get $2xc''(x) - c'(x) = 0$. This is a separable equation, and we find $c'(x) = c_1 x^{1/2}$, and $c(x) = 2/3c_1 x^{3/2} + c_2$. Hence $y_2(x) = 2/3c_1 x^{1/2} + c_2 x^{-1}$.

5. (25) Consider $y'' - 3y' - 4y = 2\sin x$.

   (a) (5) Find the general solution $y_g(x)$ for the corresponding homogeneous equation $y'' - 3y' - 4y = 0$.

   The solution is: $y_g(x) =$
   Solution. $r^2 - 3r - 4 = 0$, hence $r = -1, 4$, and $y_g(x) = c_1 e^{-x} + c_2 e^{4x}$.

(b) (15) Find a particular solution $y_p(x)$ of the equation $y'' - 3y' - 4y = 2\sin x$.

The solution is: $y_p(x) =$

Solution. We are looking for $y_p(x) = A\sin x + B\cos x$. Plugging $y_p(x)$ into the equation leads to the system of equations

$$-5A + 3B = 2, \text{ and } -3A - 5B = 0.$$

That is $y_p(x) = -\frac{5}{17}\sin x + \frac{3}{17}\cos x$.

(c) (5) Find the general solution $y(x)$ for the equation $y'' - 3y' - 4y = 2\sin x$.

The solution is: $y(x) =$

Solution. $y(x) = y_p(x) + y_g(x) = -\frac{5}{17}\sin x + \frac{3}{17}\cos x + c_1e^{-x} + c_2e^{-4x}$.

6. (25) (20) Consider $y'' + y = \frac{1}{\cos x}$, $0 < x < \frac{\pi}{2}$.

(a) (5) Find the general solution $y_g(x)$ for the corresponding homogeneous equation $y'' + y = 0$.

The solution is: $y_g(x) =$

Solution. $r^2 + 1 = 0$, hence $r = \pm i$, and $y_g(x) = c_1\cos x + c_2\sin x$.

(b) (15) Find a particular solution $y_p(x)$ of the equation $y'' + y = \frac{1}{\cos x}$.

The solution is: $y_p(x) =$

Solution. We are looking for a particular solution $y_p(x) = c_1(x)\cos x + c_2(x)\sin x$, where $c_1'(x) = -\tan x$ and $c_2'(x) = 1$. This yields

$$c_1(x) = \ln\cos x,\ c_2(x) = x,\ \text{and } y_p(x) = \cos x\ln\cos x + x\sin x.$$

(c) (5) Find the general solution $y(x)$ for the equation $y'' + y = \frac{1}{\cos x}$.

The solution is: $y(x) =$

Solution. $y(x) = y_p(x) + y_g(x) = \cos x\ln\cos x + x\sin x + c_1(x)\cos x + c_2(x)\sin x$.