

Linear Algebra Math 221 Bonus Problem Assigned 10/9/18

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Section 5

Prove: $(AB)^T = (B^T)(A^T)$

Solution: The matrix $A=(a_{ij})$ and the matrix $B=(b_{ij})$ also, $A^T=(a_{ji})$ and $B^T=(b_{ji})$.

$$AB(ij) = \sum_{k=1}^n a_{ik}b_{kj}$$

If we transpose the matrix, we switch the row and columns

$$((AB)^T)_{ij} = (AB)_{ji} = \sum_{k=1}^n a_{jk}b_{ki}$$

$$\text{Also, } ((B^T)(A^T))_{ij} = \sum_{k=1}^n ((B^T)_{ik})(A^T)_{kj} = \sum_{k=1}^n b_{ki}a_{jk}$$

This shows that the ij entry of $(AB)^T$ is equal to the ij entry of $(B^T)(A^T)$

This proves that $(AB)^T = (B^T)(A^T)$. (QED)