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Linear Algebra Proof

Given expression: t A + (1-t) B
parameters: $0 \leq t \leq 1$, A and B are each distinct points on the x-y plane.
Let A = (x1, y1) and B = (x2, y2)

My Claim: Given expression = C = line segment with endpoints A and B.

Proof: The given expression is a set of linear combinations of points A and B which will result in a set of points. C is defined as this set of points.

$$t A + (1-t) B = \{ x, y \} = C$$

Where any one point in C is calculated as (x, y) where:

$x = t (x1) + (1-t) x2$
$y = t (y1) + (1-t) y2$

In the above expression x1, x2, y1 and y2 are constants.

Notice that the variable, t, has the same relationship with x as it does with y. The only difference between calculating x and y are the given constants. Furthermore, t has a linear relationship with its constants.

With any two points in C labeled (x', y') and (x'', y''):

$(y'' - y') / (x''- x') = \text{constant}$

Thus there is a constant slope for the set of points in C and C must be a line. More specifically, C must be a line segment because t has parameters.

The endpoints of C are the extremes of the parameters of t.

For t = 0: C = B
For t = 1: C = A

Therefore C is a line segment connecting points A and B.