MATH221-04
quiz #1, 09/20/18
Total 100
Solutions

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Show all work legibly. **Name**:______________________

1. (20) Solve the following system of linear equations

$$\begin{array}{rrrl} & x_2 & -4x_3 & = 8 \\ 2x_1 & -4x_2 & +2x_3 & = 6 \\ 5x_1 & -8x_2 & +7x_3 & = 1 \end{array}$$

Solution

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -4 & 2 & 6 \\ 5 & -8 & 7 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -4 & 2 & 6 \\ 0 & 1 & -4 & 8 \\ 5 & -8 & 7 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -4 & 8 \\ 5 & -8 & 7 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -4 & 8 \\ 0 & 2 & 2 & -14 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 10 & -30 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 1 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 0 & 6 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

Mark one:

- ▫ The solutions are:
  $x_1 =$ $x_2 =$ $x_3 =$
- ▫ The system has no solutions.

2. (20) Determine the values of h for which the system

$$2x_1 + 6x_2 = -8h, \; 4x_1 + 12x_2 = h$$

is consistent.

Solution.

$$\begin{bmatrix} 2 & 6 & -8h \\ 4 & 12 & h \end{bmatrix} \to \begin{bmatrix} 2 & 6 & -8h \\ 0 & 0 & 17h \end{bmatrix}$$

$h =$

3. (20) Let $\mathbf{v}$ and $\mathbf{u}$ be two vectors of magnitude 1. If $\mathbf{v}^T\mathbf{u} = 0$ find magnitude $|\mathbf{u} + \mathbf{v}|$ of the vector $\mathbf{u} + \mathbf{v}$.

Solution.

$$|\mathbf{u} + \mathbf{v}|^2 = (\mathbf{u} + \mathbf{v})^T (\mathbf{u} + \mathbf{v}) = \mathbf{u}^T\mathbf{u} + 2\mathbf{u}^T\mathbf{v} + \mathbf{v}^T\mathbf{v} = 2.$$

4. (20) Let $\mathbf{a} \in \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$. True or False? The vector $2\mathbf{a} \in \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

Solution. If $\mathbf{a} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2$, then $2\mathbf{a} = 2c_1\mathbf{v}_1 + 2c_2\mathbf{v}_2$.

Mark one and explain.
▫ True ▫ False

5. (20) Suppose that a system of linear equations $A\mathbf{x} = \mathbf{b}$ is consistent. True or False? The system of linear equations $A\mathbf{x} = 3\mathbf{b}$ is consistent.

Solution. If vector $\mathbf{v}$ solves $A\mathbf{x} = 3\mathbf{b}$, i.e., $A\mathbf{v} = \mathbf{b}$, then $A(3\mathbf{v}) = 3\mathbf{b}$.

Mark one and explain.
▫ True ▫ False

6. (20) Let A be a 2×2 matrix so that $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, and $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Compute $A\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Solution.

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

hence

$$A\begin{bmatrix} 2 \\ 3 \end{bmatrix} = A\left(2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ -6 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}.$$