MATH 221, Spring 2018 - Homework 6 Solutions

Due Thursday, March 29

Section 4.2

Page 206, Problem 6:

Solve the equation $A\mathbf{x} = \mathbf{0}$: $\begin{bmatrix} 1 & 3 & -4 & -3 & 1 & 0 \\ 0 & 1 & -3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 5 & -6 & 1 & 0 \\ 0 & 1 & -3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Thus, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 6 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. So, a spanning set for the null space is $\left\{ \begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

Page 206, Problem 9:

The system of equations can be rearranged to $\begin{array}{c} p - 3q - 4s - 0r = 0 \\ 2p - 0q - s - 5r = 0 \end{array}$. So the vectors in W are solutions to this system.

Therefore, W is a subspace of $\mathbb{R}^4$, by Theorem 2 (and hence a vector space).

Page 206, Problem 14:

Notice that $W = Col\,\mathrm{A}$ for $A = \begin{bmatrix} -1 & 3 \\ 1 & -2 \\ 5 & -1 \end{bmatrix}$. Therefore, W is a subspce of $\mathbb{R}^3$ (and a vector space) by Theorem 3

(look at Example 4 of this section).

Page 206, Problem 27:

Let $A = \begin{bmatrix} 1 & -3 & -3 \\ -2 & 4 & 2 \\ -1 & 5 & 7 \end{bmatrix}$. Then, $\mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ is a solution to $A\mathbf{x} = \mathbf{0}$. Thus, $\mathbf{x} \in Nul\,\mathrm{A}$. Since $Nul\,\mathrm{A}$ is a subspace

of $\mathbb{R}^3$, it is closed under scalar multiplication. Therefore, $10\mathbf{x} = \begin{bmatrix} 30 \\ 20 \\ -10 \end{bmatrix}$ is also in $Nul\,\mathrm{A}$ (a solution to the system).

Page 206, Problem 28:

Let $A = \begin{bmatrix} 5 & 1 & -3 \\ -9 & 2 & 5 \\ 4 & 1 & -6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 9 \end{bmatrix}$. Because there is a solution to $A\mathbf{x} = \mathbf{b}$, $\mathbf{b} \in Col\,\mathrm{A}$. Since $Col\,\mathrm{A}$ is a subspace

of $\mathbb{R}^3$, it is closed under scalar multiplication. Thus, $5\mathbf{b} = \begin{bmatrix} 0 \\ 5 \\ 45 \end{bmatrix}$ is also in $Col\,\mathrm{A}$. So, the second system must also have a solution.

Page 207, Problem 30:

Let $T(\mathbf{x})$ and $T(\mathbf{w})$ be vectors in the range of T. Then, because T is a linear transformation, $T(\mathbf{x}) + T(\mathbf{w}) = T(\mathbf{x} + \mathbf{w})$ and for any scalar c, $cT(\mathbf{x}) = T(c\mathbf{x})$. Because $T(\mathbf{x} + \mathbf{w})$ and $T(c\mathbf{x})$ are in the range of T (which is a subset of W), it follows that the range of T is a subspace of W (closed under addition and scalar multiplication).