MATH221
quiz #1, 03/01/18
Total 120
Solutions

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Show all work legibly. **Name**:______________________

1. (20) Find a if $x_3 = 2$ and

$$\begin{array}{rrrcl} 2x_1 & & -4x_3 & = & a \\ & x_2 & +3x_3 & = & 2 \\ x_1 & +5x_2 & +8x_3 & = & 0 \end{array}$$

Solution.

$$\left\{\begin{array}{rrrcl} 2x_1 & & -8 & = & a \\ & x_2 & +6 & = & 2 \\ x_1 & +5x_2 & +16 & = & 0 \end{array}\right. \text{ and } \left\{\begin{array}{rrrcr} 2x_1 & & -a & = & 8 \\ & x_2 & & = & -4 \\ x_1 & +5x_2 & & = & -16 \end{array}\right.$$

$$\begin{bmatrix} 2 & 0 & -1 & 8 \\ 0 & 1 & 0 & -4 \\ 1 & 5 & 0 & -16 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 & -16 \\ 0 & 1 & 0 & -4 \\ 2 & 0 & -1 & 8 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 & -16 \\ 0 & 1 & 0 & -4 \\ 0 & -10 & -1 & 40 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 & -16 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$a =$

2. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 5 & 0 \end{bmatrix}$. True or False? The vector $\mathbf{v} = \begin{bmatrix} 8 \\ -4 \\ -16 \end{bmatrix}$ is a linear combination of $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$.

Solution.

$$\begin{bmatrix} 2 & 0 & -1 & 8 \\ 0 & 1 & 0 & -4 \\ 1 & 5 & 0 & -16 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 & -16 \\ 0 & 1 & 0 & -4 \\ 2 & 0 & -1 & 8 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 & -16 \\ 0 & 1 & 0 & -4 \\ 0 & -10 & -1 & 40 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 5 & 0 & -16 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & -1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\mathbf{v} = 4\mathbf{a}_1 - 4\mathbf{a}_2$.

Mark one and explain.

□ True □ False

3. (20) True or False? The matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 5 & 0 \end{bmatrix}$ is invertible.

Solution. A sequence of elementary row operations transforms A into the identity matrix (see solution for Problem 2).

Mark one and explain.

□ True □ False

4. (20) Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 5 & 0 \end{bmatrix}$. Define a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ by $T\mathbf{x} = A\mathbf{x}$.

True or False? T is onto.

Solution. If $\mathbf{b} \in \mathbf{R}^3$, then $T\mathbf{x} = \mathbf{b}$ for $\mathbf{x} = A^{-1}\mathbf{b}$.

Mark one and explain.

□ True □ False

5. (20) Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 5 & 0 \end{bmatrix}$. Define a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ by $T\mathbf{x} = A\mathbf{x}$.

True or False? T is one-to-one.

Solution. If $0 = T\mathbf{x} = A\mathbf{x}$, then $0 = A^{-1}0 = \mathbf{x}$.

Mark one and explain.

□ True □ False

6. (20) Let A be an $n \times n$ matrix so that for each $\mathbf{b} \in \mathbf{R}^n$ the system $A\mathbf{x} = \mathbf{b}$ is consistent. True or False? A^{-1} exists.

Solution. Let $\mathbf{b}_i$ be a solution for $A\mathbf{x} = \mathbf{e}_i$, that is $A\mathbf{b}_i = \mathbf{e}_i$. If $B = [\mathbf{b}_1, \ldots, \mathbf{b}_n]$, then $AB = [A\mathbf{b}_1, \ldots, A\mathbf{b}_n] = I$.

Mark one and explain.

□ True □ False