

MATH221

quiz #2, 04/05/2018

Total Possible 100

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly.

Name: _____

1. (20) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Let $H = \text{span} \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \}$. Find a basis \mathcal{B} for H .

$\mathcal{B} =$

2. (30) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) (15) Show that $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for \mathbf{R}^3 .

(b) (15) Find coordinates $[\mathbf{v}]_{\mathcal{B}}$ of the vector $\mathbf{v} = \begin{bmatrix} 6 \\ 10 \\ 9 \end{bmatrix}$ with respect to the basis \mathcal{B} .

$$[\mathbf{v}]_{\mathcal{B}} =$$

3. (20) Let $T : \mathbf{R}^n \rightarrow \mathbf{R}^n$ be an invertible linear transformation. True or False? If vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ are linearly dependent, then vectors $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_k)\}$ are linearly independent.

Mark one and explain.

- True False

4. (30) Let V be a vector space of $n \times n$ matrices. For an $n \times n$ invertible matrix A define a linear transformation $T : V \rightarrow V$ by $T(X) = AX$.

(a) (15) Describe $\ker T = \{X : X \in V, \text{ and } AX = 0\}$

$\ker T =$

(b) (15) Describe Range of T , i.e., $\{Y : Y \in V, \text{ and there is } X \in V \text{ so that } AX = Y\}$

Range $T =$

5. (20) Consider a set of three polynomials:

$$p_1(x) = 1 + x + x^2, \quad p_2(x) = 2 + 2x, \quad p_3(x) = 3 - 3x.$$

True or False? The set $\{p_1(x), p_2(x), p_3(x)\}$ is linearly dependent.

Mark one and explain.

True False