MATH221

quiz #2, 04/05/2018

Total Possible 100

Sections 2.3, 4.1-4.3

Solutions

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly. **Name**:______________________

1. (20) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Let $H = \text{span}\,\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$. Find a basis $\mathcal{B}$ for H.

Solution. Note that $\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3 = 0$, hence the vectors $\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$ are linearly dependent. On the other hand the vectors $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ are linearly independent. Hence $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ is a basis for H.

$\mathcal{B} =$

2. (30) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) (15) Show that $\mathcal{B} = \{\mathbf{v}_1, \ \mathbf{v}_2, \ \mathbf{v}_3\}$ is a basis for $\mathbf{R}^3$.

Solution. The vectors $\{\mathbf{v}_1, \ \mathbf{v}_2, \ \mathbf{v}_3\}$ are linearly independent. The matrix $V = [\mathbf{v}_1, \ \mathbf{v}_2, \ \mathbf{v}_3]$ is invertible. Hence $\mathcal{B}$ is a basis for $\mathbf{R}^3$.

(b) (15) Find coordinates $[\mathbf{v}]_{\mathcal{B}}$ of the vector $\mathbf{v} = \begin{bmatrix} 6 \\ 10 \\ 9 \end{bmatrix}$ with respect to the basis $\mathcal{B}$.

Solution. $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

3. (20) Let $T : \mathbf{R}^n \to \mathbf{R}^n$ be an invertible linear transformation. True or False? If vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ are linearly dependent, then vectors $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\}$ are linearly independent.

Solution. Let $0 = c_1\mathbf{v}_1 + \ldots + c_k\mathbf{v}_k = 0$, not all $c_i = 0$. Note that

$$0 = T\left(c_1\mathbf{v}_1 + \ldots + c_k\mathbf{v}_k\right) = T\left(c_1\mathbf{v}_1\right) + \ldots + T\left(c_k\mathbf{v}_k\right) = c_1T\left(\mathbf{v}_1\right) + \ldots + c_kT\left(\mathbf{v}_k\right).$$

Mark one and explain.

▫ True ▫ False

4. (30) Let V be a vector space of $n \times n$ matrices. For an $n \times n$ invertible matrix A define a linear transformation $T : V \to V$ by $T(X) = AX$.

(a) (15) Describe $\ker T = \{X : X \in V, \text{ and } AX = 0\}$

Solution. If $AX = 0$, then $X = A^{-1}AX = A^{-1}0 = 0$.
$\ker T =$

(b) (15) Describe Range of T, i.e., $\{Y \;:\; Y \in V, \text{ and there is } X \in V \text{ so that } AX = Y\}$

Solution. Let $Y \in V$, then $Y = AX$ with $X = A^{-1}Y$.
Range $T =$

5. (20) Consider a set of three polynomials:

$$p_1(x) = 1 + x + x^2, \; p_2(x) = 2 + 2x, \; p_3(x) = 3 - 3x.$$

True or False? The set $\{p_1(x), \; p_2(x), \; p_3(x)\}$ is linearly dependent.

Solution. The equation $c_1 p_1(x) + c_2 p_2(x) + c_3 p_3(x) = 0$ immediately yields $c_1 = 0$. We are left with $c_2 p_2(x) + c_3 p_3(x) = 0$, that is

$$\frac{1}{2}c_2 + \frac{1}{3}c_3 = 0, \text{ and } \frac{1}{2}c_2 - \frac{1}{3}c_3 = 0.$$

The only solution to the system is $c_2 = c_3 = 0$.

Mark one and explain.

□ True □ False