MATH221

quiz #3, 05/08/2018

Total Possible 100

Sections 4.4-4.6, 3.1, 3.2, 5.1-5.3

Solutions

Show all work legibly. **Name**:______________________

1. (20) Let A be a square matrix such that $\det A^2 = 0$. True or False? A is invertible.

   Solution. $0 = \det A^2 = \det A \det A$. Since $\det A = 0$ the matrix is not invertible.

   Mark one and explain.

   ▫ True ▫ False

2. (20) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 6 & 12 & 19 \end{bmatrix}$. Find $\det A$.

Solution.

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 6 & 12 & 19 \end{bmatrix} = \det \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix} = 4.$$

$\det A =$

3. (20) Let $\mathcal{B}_1 = \{\mathbf{b}_1, \mathbf{b}_2\} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \end{bmatrix} \right\}$, and $\mathcal{B}_2 = \{\mathbf{b}_1', \mathbf{b}_2'\} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$.

(a) (20) Find $[\mathbf{b}_1]_{\mathcal{B}_2}$ and $[\mathbf{b}_2]_{\mathcal{B}_2}$.

Solution. Note that $3\mathbf{b}_1' - \mathbf{b}_2' = \mathbf{b}_1$, and $-7\mathbf{b}_1' + 2\mathbf{b}_2' = \mathbf{b}_2$. Hence $[\mathbf{b}_1]_{\mathcal{B}_2} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, and $[\mathbf{b}_2]_{\mathcal{B}_2} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$.

$[\mathbf{b}_1]_{\mathcal{B}_2} =$ and $[\mathbf{b}_2]_{\mathcal{B}_2} =$

(b) (20) If $[\mathbf{v}]_{\mathcal{B}_1} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ find $[\mathbf{v}]_{\mathcal{B}_2}$.

Solution.

$$[\mathbf{v}]_{\mathcal{B}_2} = [3\mathbf{b}_1 + \mathbf{b}_2]_{\mathcal{B}_2} = 3\,[\mathbf{b}_1]_{\mathcal{B}_2} + [\mathbf{b}_2]_{\mathcal{B}_2} = \left[[\mathbf{b}_1]_{\mathcal{B}_2}, [\mathbf{b}_2]_{\mathcal{B}_2}\right] [\mathbf{v}]_{\mathcal{B}_1} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

$[\mathbf{v}]_{\mathcal{B}_2} =$

4. (20) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. If $\lambda_1 = 2$ and $\lambda_2 = 3$ are the eigenvalues of A compute $\det A$.

Solution. $\det A = \lambda_1 \lambda_2 = 6$.

$\det A =$

5. (20) Let $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. Find projection $\mathbf{p}$ of $\mathbf{w}$ on the line defined by $\mathbf{v}$.

Solution. Since $\mathbf{v}^T\mathbf{w} = 0$ one has $\mathbf{p} = 0$.

$\mathbf{p} =$