

MATH221

quiz #3, 05/08/2018

Total Possible 100

Sections 4.4-4.6, 3.1, 3.2, 5.1-5.3

Solutions

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly.

Name: _____

1. (20) Let A be a square matrix such that $\det A^2 = 0$. True or False? A is invertible.

Solution. $0 = \det A^2 = \det A \det A$. Since $\det A = 0$ the matrix is not invertible.

Mark one and explain.

- True False

2. (20) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 6 & 12 & 19 \end{bmatrix}$. Find $\det A$.

Solution.

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 6 & 12 & 19 \end{bmatrix} = \det \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix} = 4.$$

$\det A =$

3. (20) Let $\mathcal{B}_1 = \{\mathbf{b}_1, \mathbf{b}_2\} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \end{bmatrix} \right\}$, and $\mathcal{B}_2 = \{\mathbf{b}'_1, \mathbf{b}'_2\} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$.

(a) (20) Find $[\mathbf{b}_1]_{\mathcal{B}_2}$ and $[\mathbf{b}_2]_{\mathcal{B}_2}$.

Solution. Note that $3\mathbf{b}'_1 - \mathbf{b}'_2 = \mathbf{b}_1$, and $-7\mathbf{b}'_1 + 2\mathbf{b}'_2 = \mathbf{b}_2$. Hence $[\mathbf{b}_1]_{\mathcal{B}_2} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$,

and $[\mathbf{b}_2]_{\mathcal{B}_2} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$.

$[\mathbf{b}_1]_{\mathcal{B}_2} =$ and $[\mathbf{b}_2]_{\mathcal{B}_2} =$

(b) (20) If $[\mathbf{v}]_{\mathcal{B}_1} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ find $[\mathbf{v}]_{\mathcal{B}_2}$.

Solution.

$$[\mathbf{v}]_{\mathcal{B}_2} = [3\mathbf{b}_1 + \mathbf{b}_2]_{\mathcal{B}_2} = 3[\mathbf{b}_1]_{\mathcal{B}_2} + [\mathbf{b}_2]_{\mathcal{B}_2} = \begin{bmatrix} [\mathbf{b}_1]_{\mathcal{B}_2}, [\mathbf{b}_2]_{\mathcal{B}_2} \end{bmatrix} [\mathbf{v}]_{\mathcal{B}_1} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

$$[\mathbf{v}]_{\mathcal{B}_2} =$$

4. (20) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. If $\lambda_1 = 2$ and $\lambda_2 = 3$ are the eigenvalues of A compute $\det A$.

Solution. $\det A = \lambda_1 \lambda_2 = 6$.

$\det A =$

5. (20) Let $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. Find projection \mathbf{p} of \mathbf{w} on the line defined by \mathbf{v} .

Solution. Since $\mathbf{v}^T \mathbf{w} = 0$ one has $\mathbf{p} = 0$.

$\mathbf{p} =$