MATH221
quiz #1, 09/26/19
Solutions
Total 100

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Show all work legibly. **Name**:______________________

1. (20) Solve the system

$$\begin{array}{rrrcr} 2x_1 & & -4x_3 & = & 10 \\ & x_2 & +3x_3 & = & 2 \\ x_1 & +5x_2 & +8x_3 & = & 0 \end{array}$$

Solution.

$$\begin{bmatrix} 2 & 0 & -4 & 10 \\ 0 & 1 & 3 & 2 \\ 1 & 5 & 8 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 8 & 0 \\ 2 & 0 & -4 & 10 \\ 0 & 1 & 3 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 8 & 0 \\ 2 & 0 & -4 & 10 \\ 0 & 1 & 3 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 8 & 0 \\ 0 & -10 & -20 & 10 \\ 0 & 1 & 3 & 2 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 5 & 8 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 3 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 8 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 & -24 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$x_1 = 11$, $x_2 = -7$, $x_3 = 3$.

2. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 5 & 0 \end{bmatrix}$. True or False? The vector $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is a linear combination of $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$.

   Solution. $\mathbf{b} = 0\mathbf{a}_1 + 0\mathbf{a}_2 - \mathbf{a}_3$.

   Mark one and explain.
   ▫ True ▫ False

3. (20) Let $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ Describe all $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ for which the system $A\mathbf{x} = \mathbf{b}$ has a solution.

   Solution.

$$\begin{bmatrix} 1 & -1 & b_1 \\ -1 & 1 & b_2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & b_1 \\ 0 & 0 & b_1 + b_2 \end{bmatrix}$$

   When $b_1 + b_2 = 0$ the system is consistent.

4. (20) Let A be a 1×3 matrix and $\mathbf{u}$ is a vector with three entries so that $A\mathbf{u} = 1$. True or False? The equation $A\mathbf{x} = 2$ is consistent.

Solution. Note that $A(2\mathbf{u}) = 2A\mathbf{u} = 2$.

Mark one and explain.

▫ True ▫ False

5. (20) True or False? If vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent, then the vectors $\mathbf{v}_1, \mathbf{v}_2$ are also linearly independent.

Solution. If $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = 0$ and not all $x_i = 0$, then $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + 0\mathbf{v}_3 = 0$.

Mark one and explain.

▫ True ▫ False

6. (20) The matrix Let $A = \begin{bmatrix} 2 & 0 & -4 \\ 0 & 1 & 3 \\ 1 & 5 & 8 \end{bmatrix}$. True or False? All three systems

$$A\mathbf{x} = \mathbf{e}_1, \; A\mathbf{x} = \mathbf{e}_2, \; \text{ and } A\mathbf{x} = \mathbf{e}_3$$

are consistent.

Solution. A sequence of elementary row operations transforms A into the identity matrix (see solution for Problem 1).

Mark one and explain.

▫ True ▫ False