MATH221
quiz #2, 10/31/19
Solutions
Total 100

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Show all work legibly. **Name**:______________________

1. (30) Let T be a linear transformation that reflects a vector in $\mathbf{R}^2$ with respect to the line $y = -x$.

   (a) (10) Find A the standard matrix for the linear transformation T.

   Solution.
   $$A = \left[\begin{array}{cc} T(\mathbf{e}_1) & T(\mathbf{e}_2) \end{array}\right] = \left[\begin{array}{rr} 0 & -1 \\ -1 & 0 \end{array}\right]$$
   $A =$

   (b) (10) Find A^{-1} if exists.

   Solution. $A^{-1} = A$.
   $A^{-1} =$

   (c) (10) True or False? T is one–to–one.

   Solution. If $0 = T(\mathbf{x}) = A\mathbf{x}$, then $0 = A^{-1}0 = \mathbf{x}$.
   Mark one and explain.
   ▫ True ▫ False

2. (20) Let $p_1(x) = 1 + x + x^2$, $p_2(x) = 1$, and $p_3(x) = x - x^2$.

   (a) (10) True or False? The vector set $\{p_1(x), p_2(x), p_3(x)\}$ is linearly independent.

   Solution. If
   $$c_1p_1(x) + c_2p_2(x) + c_3p_3(x) = 0 \quad (1)$$
   then
   $$0 = c_1p_1'(x) + c_2p_2'(x) + c_3p_3'(x) = c_1[1 + 2x] + c_3[1 - 2x].$$
   An additional differentiation leads to $c_1 - c_3 = 0$. The equation above then yields $c_1 = c_3 = 0$. Finally due to (1) one has $c_2 = 0$.
   Mark one and explain.
   ▫ True ▫ False

(b) (10) True or False? The vector set $\{p_1(x), p_2(x), p_3(x)\}$ spans P_2.

Solution. Note that the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ is invertible.

Mark one and explain.
▫ True ▫ False

3. (20) Let $A = \mathbf{u}\mathbf{v}^T$, where $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$.

(a) (10) Find a basis size for Nul(A).

Solution. Note that $0 = A\mathbf{x} = \mathbf{u}(\mathbf{v}^T\mathbf{x})$. Since $\mathbf{u}$ is a the condition $\mathbf{x} \in \text{Nul}(A)$ yields $0 = \mathbf{v}^T\mathbf{x}$, i.e., $\mathbf{x} \in \text{Nul}(\mathbf{v}^T)$. Finally a basis size for Nul(A) = a basis size for $\text{Nul}(\mathbf{v}^T) = 2$.

a basis size for Nul(A) =

(b) (10) Find a basis size for Col(A).

Solution. Since a basis size for Col(A)+ a basis size for Nul(A) = 3, one has a basis size for Col(A) = 1.

a basis size for Col(A) =

4. (10) Let $A = \begin{bmatrix} 2 & 0 & -4 \\ 0 & 1 & 3 \\ 1 & 5 & 8 \end{bmatrix}$. Find Nul($A$).

Solution.

$$\begin{bmatrix} 2 & 0 & -4 \\ 0 & 1 & 3 \\ 1 & 5 & 8 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 8 \\ 2 & 0 & -4 \\ 0 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 8 \\ 0 & -10 & -20 \\ 0 & 1 & 3 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Nul(A) is

5. (20) Let $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ and $B' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ be two bases in $\mathbf{R}^3$. If $[\mathbf{u}]_B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ find $[\mathbf{u}]_{B'}$.

Solution. $[\mathbf{u}]_{B'} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

$[\mathbf{u}]_{B'} =$

6. (20) True or False? The vector set $\{\sin t, \sin 2t, \sin 3t\}$ is linearly independent.

Solution. Let $c_1 \sin t + c_2 \sin 2t + c_3 \sin 3t = 0$ for each real t. For $t = \frac{\pi}{2}$ this becomes

$$c_1 + 0c_2 + c_3 = 0.$$

The differentiation of both sides of the identity $c_1 \sin t + c_2 \sin 2t + c_3 \sin 3t = 0$ leads to $c_1 \cos t + 2c_2 \cos 2t + 3c_3 \cos 3t = 0$ for each real t. For $t = 0$ one has

$$c_1 + 2c_2 + 3c_3 = 0,$$

and for $t = \frac{\pi}{2}$ the identity becomes

$$0c_1 - 2c_2 + 0c_3 = 0.$$

Finally $c_1 = c_2 = c_3 = 0$.

Mark one and explain.
□ True □ False