MATH221
quiz #3, 12/03/19
Solutions
Total 100

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Show all work legibly. **Name**:______________________

1. (20) Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$. Compute $\det A$

   Solution.
   $$\det A = \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & 2 & 12 \end{bmatrix} = \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{bmatrix} = 2.$$

   $\det A =$

2. (40) Let $A = \begin{bmatrix} 2 & -2 & -2 \\ 3 & -3 & -2 \\ 2 & -2 & -2 \end{bmatrix}$ with the eigenvalues $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -2$.

   (a) (20) Find eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ corresponding to the eigenvalues λ_1, λ_2, and λ_3.

   Solution. A solution for $(A - \lambda_1)\mathbf{x} = 0$ is $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

   A solution for $(A - \lambda_2)\mathbf{x} = 0$ is $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$.

   A solution for $(A - \lambda_3)\mathbf{x} = 0$ is $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

   $\mathbf{v}_1 =$, $\mathbf{v}_2 =$, $\mathbf{v}_3 =$

(b) (10) Let $V = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$. Find V^{-1} if exists.

Solution.

$$\left[\begin{array}{cccccc} -1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & -2 & -1 & -1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \to$$

$$\left[\begin{array}{cccccc} 1 & -2 & -1 & -1 & 0 & 0 \\ 0 & 1 & 2/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & -1/3 & -2/3 & -2/3 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & -2 & -1 & -1 & 0 & 0 \\ 0 & 1 & 2/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1 & 2 & 2 & -3 \end{array}\right] \to$$

$$\left[\begin{array}{cccccc} 1 & -2 & 0 & 1 & 2 & -3 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & 2 & 2 & -3 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & 2 & 2 & -3 \end{array}\right]$$

$V^{-1} =$

(c) (10) Compute A^{10}.

Solution. $A^{10} = VD^{10}V^{-1}$, where

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \; V = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \text{ and } V^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$

$A^{10} =$

3. (20) Let V be a subspace of $\mathbf{R}^3$ with an orthonormal basis $\{\mathbf{v}_1, \mathbf{v}_2\}$, and $\mathbf{w}$ is a vector of norm 1 in $V^{\perp}$. True or False? The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{w}\}$ is a basis for $\mathbf{R}^3$.

Solution. Let $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c\mathbf{w} = 0$. Dot multiplication of the left hand side and the right hand side with $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{w}$ yields $c_1 = c_2 = c = 0$. This shows that the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{w}\}$ is linearly independent, hence a basis for $\mathbf{R}^3$.

Mark one and explain.

□ True □ False

4. (20) Let $V = \text{span}\,\{\mathbf{v}_1, \mathbf{v}_2\}$ where $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$. For $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ find the orthogonal projection of $\mathbf{w}$ onto V.

Solution. Since $\mathbf{w}$ is in V the orthogonal projection is $\mathbf{w}$.

the orthogonal projection is

5. (20) Use the basis $\{\mathbf{v}_1, \mathbf{v}_2\}$ for $\mathbf{R}^2$ with $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ to build an orthonormal basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for $\mathbf{R}^2$.

Solution. Since $\mathbf{v}_1^T \mathbf{v}_2 = 0$ a simple possible choice of an orthonormal basis is $\mathbf{u}_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$.

$\mathbf{u}_1 =$ $\mathbf{u}_2 =$