

## Homework 5

1. Let  $A \in \mathbf{R}^{n \times n}$ . For  $k \geq 0$  denote  $\text{span} \left\{ I, A, A^2, \dots, A^k \right\}$  by  $V_k(A)$ .
  - (a) Show that there is  $0 \leq k \leq n \times n$  so that  $V_k(A) = V_{k+i}(A)$ ,  $i = 1, 2, \dots$
  - (b) The vectors  $\{I, A, \dots, A^k\}$  are linearly independent.
  - (c) We denote the matrix set  $\{\mathbf{v}_i \mathbf{v}_j : \mathbf{v}_i \in V_i(A), \mathbf{v}_j \in V_j(A)\}$  by  $V_i(A) \times V_j(A)$ . Show that  $V_{k+i} \times V_{k+j} = V_k$ .
  - (d) If  $A^{-1}$  exists, then  $V_k(A) = V_k(A^{-1})$ .