

Homework 5

1. Let $A \in \mathbf{R}^{n \times n}$. For $k \geq 0$ denote $\text{span} \{I, A, A^2, \dots, A^k\}$ by $V_k(A)$.
 - (a) Show that there is $0 \leq k \leq n \times n$ so that $V_k(A) = V_{k+i}(A)$, $i = 1, 2, \dots$
 - (b) The vectors $\{I, A, \dots, A^k\}$ are linearly independent.
 - (c) We denote the matrix set $\{\mathbf{v}_i \mathbf{v}_j : \mathbf{v}_i \in V_i(A), \mathbf{v}_j \in V_j(A)\}$ by $V_i(A) \times V_j(A)$. Show that $V_{k+i} \times V_{k+j} = V_k$.
 - (d) If A^{-1} exists, then $V_k(A) = V_k(A^{-1})$.