Homework 3 Solutions

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4) Linear Coeffecients

29) $a_1 = -3 \quad b_1 = 1 \qquad a_2 = 1 \quad b_2 = 1$

$$a_1 b_2 = -3 \neq 1 = a_2 b_1$$

Use translational axis $\quad x = u + h, \ y = v + k$

$$-3h + k = 1 \qquad h = -1$$
$$h + k = -3 \qquad k = -2$$

$x = u - 1 \qquad y = v - 2$

$dx = du \qquad dy = dv$

$$(-3(u-1) + (v-2) - 1)\,du + ((u-1) + (v-2) + 3)\,dv = 0$$

$$(-3u + v)\,du + (u + v)\,dv = 0$$

$$\frac{dv}{du} = -\frac{(-3u+v)}{(u+v)} = \frac{3 - v/u}{1 + v/u}$$

let $z = v/u \qquad v = zu \qquad \dfrac{dv}{du} = z + u\dfrac{dz}{du}$

$$z + u\frac{dz}{du} = \frac{3-z}{1+z} \qquad u\frac{dz}{du} = -\frac{z^2+2z-3}{1+z}$$

$$u\,dz = -\frac{(z^2+2z-3)\,du}{1+z}$$

$$-\int \frac{1+z}{z^2+2z-3}\,dz = \int \frac{1}{u}\,du$$

$w = z^2 + 2z - 3 \qquad dw = 2z + 2$

$$-\frac{1}{2}\int \frac{dw}{w} = \ln u + C$$

$$\ln|z^2+2z-3| = -2\ln|u| + C$$

$$z^2 + 2z - 3 = \frac{C}{u^2}$$

$$\frac{v^2}{u^2} + 2\frac{v}{u} - 3 = \frac{C}{u^2}$$

$$v^2 + 2vu - 3u^2 = C$$

$$(y+2)^2 + 2(y+2)(x+1) - 3(x+1)^2 = C$$

31) $a_1 = 2 \quad b_1 = -1 \quad c_1 = 0$

$a_2 = 4 \quad b_2 = 1 \quad c_2 = -3$

$a_1 b_2 = 2 \neq -4 = a_2 b_1$

Use translational axes

$x = u + h \qquad y = v + K$

$2h - K = 0 \qquad h = 1/2$

$4h + K = 3 \qquad K = 1$

$x = u + 1/2 \quad y = v + 1$

$dx = du \qquad dy = dv$

$(2(u + 1/2) - (v+1))du + (4(u + 1/2) + v + 1 - 3)dv = 0$

$(2u - v)du + (4u + v)dv = 0$

$$\frac{dv}{du} = -\frac{(2u - v)}{4u + v} = \frac{-2 + v/u}{4 + v/u} \qquad z = \frac{v}{u} \qquad v = zu$$

$$\frac{dv}{du} = z + u\frac{dz}{du}$$

$$z + u\frac{dz}{du} = \frac{-2 + z}{4 + z}$$

$$u\frac{dz}{du} = \frac{-z^2 - 3z - 2}{4 + z} = -\frac{z^2 + 3z + 2}{4 + z}$$

$$\frac{du}{u} = -\int \frac{4 + z}{(z+2)(z+1)} dz$$

$$\frac{a}{z+2} + \frac{b}{z+1} \qquad az + a + bz + 2b = 4 + z$$

$a + b = 1 \qquad b = 3$

$a + 2b = 4 \qquad a = -2$

$$\ln|u| = -\int \left(\frac{-2}{z+2} + \frac{3}{z+1}\right) dz$$

$$\ln|u| = 2\ln|z+2| - 3\ln|z+1| + C$$

$$u = C\frac{(z+2)^2}{(z+1)^3} \qquad (z+1)^3 u = C(z+2)^2$$

$$\left(\frac{v}{u}+1\right)^3 u = C\left(\frac{v}{u}+2\right)^2$$

$$(v+u)^3 \frac{\cancel{u}}{\cancel{u^3}} = C(v+2u)^2 \frac{1}{\cancel{u^2}}$$

$$(2y+2x-3)^3 = C(y+2x-2)^2$$

32) $a_1 = 2 \quad b_1 = 1 \quad c_1 = 4$

$a_2 = 1 \quad b_2 = -2 \quad c_2 = -2$

$2h + k = -4 \qquad h = -6/5$

$h - 2k = 2 \qquad k = -8/5$

$(2u+v)\,du + (u-2v)\,dv = 0$

$$\frac{dv}{du} = -\frac{2u+v}{u-2v} \qquad z = v/u \quad zu = v$$

$$\frac{dv}{du} = z + u\frac{dz}{du}$$

$$z + u\frac{dz}{du} = -\frac{2+z}{1-2z}$$

$$u\frac{dz}{du} = \frac{2z^2-2z-2}{1-2z}$$

$$\int \frac{du}{u} = \frac{1}{2}\int \frac{1-2z}{z^2-z-1}\,dz \qquad w = z^2-z-1$$

$$dw = (2z-1)\,dz$$

$$\ln|u| = -\frac{1}{2}\int \frac{1}{w}\,dw$$

$$-2\ln|u| = \ln|w| + C$$

$$\frac{1}{u^2} = C(z^2-z-1)$$

$$C = \left(z^2 - z - 1\right)u^2$$

$$C = \left(\frac{v^2}{\cancel{u^2}}\cdot \cancel{u^2} - \frac{v}{\cancel{u}}\cdot u^{\cancel{2}} - u^2\right)$$

$$C = (5y+8)^2 - (5x+6)(5y+8) - (5x+6)^2$$

42) $\frac{dy}{dx} = 2xv + x^2\frac{dv}{dx}$

$2xv + x^2\frac{dv}{dx} = \frac{2y}{x} + \cos v$

$\frac{2x\cancel{y}}{\cancel{x^2}} + x^2\frac{dv}{dx} = \frac{2y}{\cancel{x}} + \cos v$

$\int \frac{dv}{\cos v} = \int \frac{dx}{x^2}$

$\ln|\tan v + \sec v| = -x^{-1} + C$

$\ln|\tan y/x^2 + \sec y/x^2| = -x^{-1} + C$