Homework 4 Solutions

Scott Dusek

7, 10, 13, 15, 20, 28, 34

7) $6m^2 + m - 2 = 0$

$$\frac{-1 \pm \sqrt{1+48}}{12} = \frac{-1 \pm 7}{12} = \frac{-8}{12}, \frac{6}{12}$$

$$y = C_1 e^{-2/3\, t} + C_2 e^{t/2}$$

10) $m^2 - m - 11 = 0$ $\qquad \dfrac{1 \pm \sqrt{1+44}}{2} = \dfrac{1 \pm \sqrt{45}}{2}$

$$y = C_1 e^{(1+\sqrt{45})/2\, t} + C_2 e^{(1-\sqrt{45})t/2}$$

13) $m^2 + 2m - 8 = 0$ $\qquad (m+4)(m-2) = 0$ $\qquad m = -4, 2$

$y = C_1 e^{-4t} + C_2 e^{2t}$ $\qquad y' = -4C_1 e^{-4t} + 2C_2 e^{2t}$

$y(0) = C_1 + C_2 = 3$ $\qquad y'(0) = -4C_1 + 2C_2 = -12$

$C_2 = 0$ $\qquad C_1 = 3$

$y(t) = 3e^{-4t}$

15) $m^2 - 4m - 5 = 0$ $\qquad \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{16+20}}{2} = \dfrac{4 \pm 6}{2} = 5, -1$

$y(t) = C_1 e^{5t} + C_2 e^{-t}$ $\qquad y'(t) = 5C_1 e^{5t} - C_2 e^{-t}$

$y(-1) = C_1 e^{-5} + C_2 e = 3$ $\qquad y'(-1) = 5C_1 e^{-5} - C_2 e = 9$

$6C_1 e^{-5} = 12$ $\qquad C_1 = 2e^{5}$

$y(-1) = 2e^{5} e^{-5} + C_2 e = 3$ $\qquad C_2 = e^{-1}$

$y(t) = 2e^{5} e^{5t} + e^{-1} e^{-t}$

20) $m^2 - 4m + 4 = 0$ $\qquad (m-2)(m-2) = 0$ $\qquad m = 2, 2$

$y(t) = C_1 e^{2t} + C_2 t e^{2t}$ $\qquad y'(t) = 2C_1 e^{2t} + C_2 e^{2t} + 2C_2 t e^{2t}$

$y(1) = C_1 e^2 + C_2 e^2 = 1$ $\qquad y'(1) = 2C_1 e^2 + C_2 e^2 + 2C_2 e^2 = 2C_1 e^2 + 3C_2 e^2 = 1$

$C_2 = -1e^{-2}$ $\qquad C_1 = 2e^{-2}$

$y(t) = 2e^{-2} e^{2t} - e^{-2} t e^{2t}$

25) linearly independent

3a) $W[y_1, y_2](t) = \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix} = y_1 y_2' - y_2 y_1'$

b) Suppose y_1 and y_2 are linearly independent and the Wronskian is 0. Then $y_1 y_2' + y_1' y_2 = 0$, and by lemma 1 y_1 and y_2 are linearly dependent. This contradicts the assumption they are lin ind, Therefore, if y_1 and y_2 are lin ind, then the Wronskian is never zero on the Interval I.

c) Let $y_1(t)$ and $y_2(t)$ be 2 differentiable functions, such that they are lin dependent. Then, $y_1(t) = C y_2(t)$ by definition of lin dependent. So, $y_1'(t) = C y_2'(t)$
Then $W = \begin{vmatrix} y_1(t) & C y_1(t) \\ y_1'(t) & C y_1'(t) \end{vmatrix} = C y_1 y_1' - C y_1 y_1' = 0$. Q.E.D.