4.3 3, 8, 24, 31(a) 4.6 1, 4, 7, 12, 17, 18

4.4 9, 11, 20, 23, 26

4.5 1, 3, 5, 8, 19, 20, 29, 41 Solutions

3) $m^2 - 10m + 26 = 0$ $\frac{10 \pm \sqrt{100-104}}{2} = 5 \pm i$

$y = C_1 e^{5t} \sin t + C_2 e^{5t} \cos t$

8) $4m^2 - 4m + 26 = 0$ $\frac{4 \pm \sqrt{16-416}}{8} = 1/2 \pm 20i/8$

$y = C_1 e^{t/2} \sin 5t/2 + C_2 e^{t/2} \cos 5t/2$

24) $m^2 + 9 = 0$ $m = \pm 3i$

$y = C_1 \sin 3t + C_2 \cos 3t$ $y' = 3C_1 \cos 3t - 3C_2 \sin 3t$

$y(0) = C_2 = 1$ $y'(0) = 3C_1 = 1$ $C_1 = 1/3$

$y(t) = 1/3 \sin 3t + \cos 3t$

31) $m^2 + 16 = 0$ $m = \pm 4i$

$y = C_1 \cos 4t + C_2 \sin 4t$ $y' = -4C_1 \sin 4t + 4C_2 \cos 4t$

$y(0) = C_1 = 2$ $y'(0) = 4C_2 = 0$ $C_2 = 0$

$y = 2\cos 4t$ This oscillates forever

4.4

9) Guess $y_p = At^2 + Bt + C$ $y_p' = 2At + B$ $y_p'' = 2A$

$2A + 2(2At + B) - At^2 - Bt - C = 10$

$-At^2 = 0 \Rightarrow A = 0$

$4At - Bt = 0 \Rightarrow B = 0$

$2A + 2B - C = 10 \Rightarrow C = -10$

$y_p = -10$

11) Guess $y_p = A2^x$ $y_p' = A\ln 2\, 2^x$ $y_p'' = A(\ln 2)^2 2^x$

$A(\ln 2)^2 2^x + A2^x = 2^x$

$A((\ln 2)^2 + 1)2^x = 2^x$

$A = \dfrac{1}{(\ln 2)^2 + 1}$ $y_p = \dfrac{2^x}{(\ln 2)^2 + 1}$

20) $m^2 + 4 = 0$ $m = \pm 2i$

$y_h = C_1 \sin 2t + C_2 \cos 2t$

Guess

$y_p = (At^2 + Bt)\sin 2t + (Ct^2 + Dt)\cos 2t$

$y_p' = (2At + B)\sin 2t + 2(At^2 + Bt)\cos 2t + (2Ct + D)\cos 2t$
$- 2(Ct^2 + Dt)\sin 2t$

$y_p'' = (2A)\sin 2t + 2(2At + B)\cos 2t + 2(2At + B)\cos 2t$
$- 4(At^2 + Bt)\sin 2t + (2C)\cos 2t - 2(2Ct + D)\sin 2t$
$- 2(2Ct + D)\sin 2t - 4(Ct^2 + Dt)\cos 2t = 16t\sin 2t$

$C = -2$ $B = 1$

$y_p = -2t^2 \cos 2t + t\sin 2t$

23) $m^2 - 7m = 0$ $m(m-7) = 0$

$m = 0, 7$ $y = C_1 + C_2 e^{7\theta}$

Guess

$y_p = A\theta^3 + B\theta^2 + C\theta$ $y_p' = 3A\theta^2 + 2B\theta + C$

$y_p'' = 6A\theta + 2B$

$6A\theta + 2B - 7(3A\theta^2 + 2B\theta + C) = \theta^2$

$-21A\theta^2 = \theta^2$ $2B - 7C = 0$

$6A\theta - 14B\theta = 0$

$A = -1/21$ $B = -1/49$ $C = -2/343$

$y_p = \dfrac{-1}{21}\theta^3 - \dfrac{1}{49}\theta^2 - \dfrac{2}{343}\theta$

26) $m^2+2m+2=0$ $\quad \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

$y_h = C_1 e^{-t}\cos t + C_2 e^{-t}\sin t$

Guess

$y_p = (At^2+Bt)e^{-t}\cos t + (Ct^2+Dt)e^{-t}\sin 2t$

$y_p' = e^{-t}(-At^2+2At)\cos t + e^{-t}(-Bt+B)\cos x - e^{-t}(Ax^2+Bx)\sin x$

$y_p'' = -2e^{-t}(2At - At^2 - Bt + B)\sin t + e^{t}(2At - A + B)\cos t$

$B=0 \quad C=1 \quad D=2 \qquad y = te^{-t}\cos t + e^{-t}t^2 \sin 2t$

1) a) $y = \frac{\sin 2t}{4} + \frac{t}{4} - \frac{1}{8}$

b) $y = \frac{3\sin 2t}{4} + \frac{t}{2} - \frac{1}{4}$

c) $y = -3\sin 2t + \frac{11t}{4} - \frac{11}{8}$

3) $m^2+m=0 \qquad m(m+1)=0 \qquad m=0,-1$

$y_h = C_1 + C_2 e^{-t}$

Guess $\quad y_p = At^2+Bt \qquad y_p' = 2At+B \qquad y_p'' = 2A$

$2A + 2At + B = 1 \qquad 2At = 0 \qquad A=0$

$2A + B \qquad B = 1$

$y_p = 1 \qquad y = C_1 + C_2 e^{-t} + 1$

5) $m^2+5m+6=0 \qquad \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2} \qquad m = -3, -2$

$y = C_1 e^{-3x} + C_2 e^{-2x} + e^{x} + x^2$

8) $m^2-2m+1=0$ $\quad m=1$

$y=C_1e^x+C_2xe^x+x^2e^x$

19) $m^2-3m+2=0$

$(m-2)(m-1)$ $\quad m=2,1$

$y_h=C_1e^x+C_2e^{2x}$

Guess

$y_p=Ae^x\sin x+Be^x\cos x$ $\quad y_p'=Ae^x\sin x+Ae^x\cos x+Be^x\cos x$
$-Be^x\sin x$

$y_p''=Ae^x\sin x+Ae^x\cos x+Ae^x\cos x-Ae^x\sin x+Be^x\cos x-Be^x\sin x$
$-Be^x\sin x-Be^x\cos x=2Ae^x\cos x-2Be^x\sin x$

$2A\cos x-2B\sin x-3(A-B)\sin x-3(A+B)\cos x+2A\sin x+2B\cos x$
$=\sin x$

$-3A-2B+3B+2A=1$ $\quad B-A=1$

$2A-3A-3B+2B$ $\quad -A-B=0$

$A=-1/2$ $\quad B=1/2$

$y=C_1e^x+C_2e^{2x}+\frac{e^x\cos x}{2}-\frac{e^x\sin x}{2}$

20) $m^2+4=0$ $\quad m=\pm 2i$

$y_h=C_1\sin 2\theta+C_2\cos 2\theta$

Guess $\quad y_p=A\sin\theta+B\cos\theta$ $\quad y_p'=A\cos\theta-B\sin\theta$

$y_p''=-A\sin\theta-B\cos\theta$

$-A\sin\theta-B\cos\theta+4A\sin\theta+4B\cos\theta=\sin\theta-\cos\theta$

$3A=1$ $\quad 3B=-1$

$A=1/3$ $\quad B=-1/3$

$y=C_1\sin 2\theta+C_2\cos 2\theta+1/3\sin\theta-1/3\cos\theta$

29) $m^2 - 1 = 0$ $m = \pm 1$ $y_h = C_1 e^t + C_2 e^{-t}$

Guess $y_p = A\sin\theta + B\cos\theta + Ce^{2\theta}$

$y_p' = A\cos\theta - B\sin\theta + 2Ce^{2\theta}$ $y_p'' = -A\sin\theta - B\cos\theta + 4Ce^{2\theta}$

$-A\sin\theta - B\cos\theta + 4Ce^{2\theta} - A\sin\theta - B\cos\theta - Ce^{2\theta} = \sin\theta - e^{2\theta}$

$-2A = 1$ $-2B = 0$ $3C = -1$

$A = -1/2$ $C = -1/3$

$y = -1/2\sin\theta - 1/3 e^{2\theta} + C_1 e^t + C_2 e^{-t}$ $y(0) = -1/3 + C_1 + C_2 = 1$

$y' = -1/2\cos\theta - 2/3 e^{2\theta} + C_1 e^t - C_2 e^{-t}$ $y'(0) = -1/2 - 2/3 + C_1 - C_2 = -1$

$C_1 = 3/4$ $C_2 = 7/12$

$y = 3/4 e^{\theta} + 7/12 e^{-\theta} - 1/2\sin\theta - 1/3 e^{2\theta}$

41) $m^2 + 2m + 5$ $\frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$

$y_h = C_1 e^{-t}\sin 2t + C_2 e^{-t}\cos 2t$

$y_p = A$ $y_p' = 0$ $y_p'' = 0$

$0 + 20 + 5A = 10$ $A = 2$

$t < 3\pi/2$ $y = C_1 e^{-t}\sin 2t + C_2 e^{-t}\cos 2t + 2$, $y(0) = C_2 + 2 = 0$ $C_2 = -2$

$y' = -C_1 e^{-t}\sin 2t + C_1 e^{-t}\cos 2t - C_2 e^{-t}\cos 2t - 2C_2 e^{-t}\sin 2t$

$y'(0) = C_1 - C_2 = 0$ $C_1 = -2$

$y = -2e^{-t}\sin 2t - 2e^{-t}\cos 2t + 2$ for $0 < t < 3\pi/2$

b) If $y'' + 2y' + 5 = 0$, then $y = y_h = C_1' e^{-t}\sin 2t + C_2' e^{-t}\cos 2t$

c) At $3\pi/2$, they must agree, so

$-2e^{-3\pi/2}\sin 3\pi - 2e^{-3\pi/2}\cos 3\pi + 2 = C_1' e^{-3\pi/2}\sin 3\pi + C_2' e^{-3\pi/2}\cos 3\pi$

$-C_2' = 2 + 2e^{3\pi/2}$ $C_2' = -2(1 + e^{3\pi/2})$

$-C_1' e^{-3\pi/2}\cancel{\sin 3\pi} + 2C_1' e^{-3\pi/2}\cos 3\pi - C_2' e^{-3\pi/2}\cos 3\pi - 2C_2' e^{-3\pi/2}\cancel{\sin 3\pi}$

$= -C_1 e^{-3\pi/2}\cancel{\sin 3\pi} + C_1 e^{-3\pi/2}\cos 3\pi - C_2 e^{-3\pi/2}\cos 3\pi - 2C_2 e^{-3\pi/2}\cancel{\sin 3\pi}$

$-2C_1' e^{-3\pi/2} + C_2' e^{-3\pi/2} = -C_1 e^{-3\pi/2} + C_2 e^{-3\pi/2}$

$C_1' = -(e^{3\pi/2} + 1)$

1) $m^2+1=0 \qquad m=\pm i$

$y_h = C_1 \sin t + C_2 \cos t$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \sin t & \cos t \\ \cos t & -\sin t \end{vmatrix} = -(\sin^2 t + \cos^2 t)$

$= -1$

$v_1' = -\frac{\sec t \cos t}{-1} \qquad v_2' = \frac{\sec t \sin t}{-1}$

$v_1' = 1 \qquad v_2' = -\tan t$

$v_1 = t \qquad v_2 = \ln|\cos t|$

$y_g = C_1 \sin t + C_2 \cos t + t\sin t + \ln|\cos t| \cdot \cos t$

4) $m^2 - 2m + 1 = (m-1)^2$

$y_h = C_1 e^t + C_2 te^t$

$y_1 = e^t \qquad y_2 = te^t$

$y_1' = e^t \qquad y_2' = e^t + te^t$

$W = \begin{vmatrix} e^t & te^t \\ e^t & e^t + te^t \end{vmatrix} = e^{2t} + te^{2t} - te^{2t} = e^{2t}$

$v_1' = -\frac{f(t) y_2}{W} \qquad v_2' = \frac{f(t) y_1}{W}$

$v_1' = -\frac{t^{-1} e^t te^t}{e^{2t}} \qquad v_2' = \frac{t^{-1} e^t e^t}{e^{2t}}$

$= -1 \qquad = \frac{1}{t}$

$v_1 = -t \qquad v_2' = \ln|t|$

$y = C_1 e^t + C_2 te^t - te^t + te^t \ln|t|$

$= C_1 e^t + C_2 te^t + te^t \ln|t|$

7) $m^2+4m+4=0$ $(m+2)(m+2)$ $m=2$

$y_h = C_1 e^{-2t} + C_2 t e^{-2t}$

$y_1 = e^{-2t}$ $y_2 = te^{-2t}$

$y_1' = -2e^{-2t}$ $y_2' = e^{-2t} - 2te^{-2t}$

$W = e^{-4t} - 2te^{-4t} + 2te^{-4t} = e^{-4t}$

$v_1' = \dfrac{-e^{-2t}\ln t \, te^{-2t}}{e^{-4t}} = -t\ln t$ $v_2' = \dfrac{\ln t \, e^{-2t} e^{-2t}}{e^{-4t}} = \ln t$

$v_1 = \dfrac{1}{4} t^2(1-2\ln|t|)$ $v_2 = t\ln|t| - t$

$y = C_1 e^{-2t} + C_2 te^{-2t} + (t\ln|t| - t)te^{-2t} + \dfrac{1}{4} t^2\left(1-2\ln|t|\right)e^{-2t}$

12) $m^2+1=0$ $m = \pm i$

$y_h = C_1 \sin t + C_2 \cos t$ $y_1 = \sin t$ $y_2 = \cos t$

$y_1' = \cos t$ $y_2' = -\sin t$

$W = -1$

$v_1 = \int \dfrac{-(\tan t + e^{3t} - 1)\cos t}{-1}\, dt$ $v_2 = -\int (\tan t + e^{3t} - 1)\sin t \, dt$

$v_1 = -\cos t + \dfrac{1}{10} e^{3t}(\sin t + 3\cos t) - \sin t$

$v_2 = -\ln|\tan x + \sec x| + \sin x + \dfrac{1}{10} e^{3t}(\cos t - 3\sin t) - \cos t$

$y = C_1 \sin x + C_2 \cos x - \ln|\sec x + \tan x|\cos x + \dfrac{1}{10} e^{3t} - 1$

Don't do this use Undetermined Coeffecient for e^{3t} and -1

17) $\frac{1}{2}m^2 + 2 = 0$ $m^2+4=0$ $m = \pm 2i$

$y_h = C_1 \sin 2t + C_2 \cos 2t$ Guess $y_p = Ae^t$ $y_p' = Ae^t$ $y_p'' = Ae^t$

$A/2\, e^t + 2Ae^t = -1/2\, e^t$ $5/2 A = -1/2$

$A = -1/5$

$y_1 = \sin 2t \qquad y_2 = \cos 2t$
$y_1' = 2\cos 2t \qquad y_2' = -2\sin 2t$

$W = -2$

$v_1 = -\int \frac{\tan 2t \cos 2t}{-2}\, dt \qquad v_2 = \frac{-1}{2}\int \tan 2t \sin 2t \, dt$

$v_1 = -1/2 \cos^2 t \qquad v_2 = \frac{\ln|\sec 2t + \tan 2t|}{2}$

$y = C_1 \sin 2t + C_2 \cos 2t - \frac{e^t}{5} - \frac{1}{2}\cos 2t \ln\left|\sec 2t + \tan 2t\right|$

18) $m^2 - 6m + 9 = 0 \qquad (m-3)(m-3)$

$y_h = C_1 e^{3t} + C_2 t e^{3t}$

$y_1 = e^{3t} \qquad y_2 = te^{3t}$
$y_1' = 3e^{3t} \qquad y_2' = e^{3t} + 3te^{3t}$

$W = e^{6t} + 3te^{6t} - 3te^{6t} = e^{6t}$

$v_1 = -\int \frac{t^{-3}e^{3t}te^{3t}}{e^{6t}}\, dt \qquad v_2 = \int \frac{t^{-3}e^{3t}e^{3t}}{e^{6t}}\, dt$

$= \frac{1}{t} \qquad v_2 = \frac{-t^{-2}}{2}$

$y = C_1 e^{3t} + C_2 t e^{3t} + \frac{1}{2t} e^{3t}$