15) $M = \cos\theta$ $\quad$ $N = -r\sin\theta + e^{\theta}$

$\frac{\partial M}{\partial \theta} = -\sin\theta$ $\quad$ $\frac{\partial N}{\partial r} = -\sin\theta$

Exact

$F(r,\theta) = \int M(r,\theta) + g(\theta)$

$\frac{\partial F}{\partial \theta} = \frac{\partial}{\partial \theta}\int M(r,\theta)dr + g'(\theta)$

$-r\sin\theta + e^{\theta} = \frac{\partial}{\partial \theta} r\cos\theta + g'(\theta)$

$\cancel{-r\sin\theta} + e^{\theta} = \cancel{-r\sin\theta} + g'(\theta)$

$g'(\theta) = e^{\theta}$ $\quad$ $g(\theta) = e^{\theta}$

$F = r\cos\theta + C + e^{\theta}$

21) $M = 1/x + 2y^2x$ $\quad$ $N = 2yx^2 - \cos y$

$F = \int (1/x + 2y^2x)dx + g(y)$

$N = \frac{\partial}{\partial y}\left(\ln x + y^2x^2\right) + g'(y)$

$\cancel{2yx^2} - \cos y = \cancel{2yx^2} + g'(y)$ $\quad$ $g'(y) = -\cos y$ $\quad$ $g(y) = -\sin(y)$

$F = \ln x + y^2x^2 + C - \sin(y)$ $\quad$ $\ln x$

$y(1) = \pi$ $\quad$ $\ln x + x^2y^2 - \sin y = \pi^2$