Homework #2 2.6 1, 3, 11, 13, 21, 28 Solutions

Scott Dusek

1) Homogeneous $\frac{dx}{dt} = \frac{-(t^2-x^2)}{2tx}$

Bernoulli: $\frac{dx}{dt} = \frac{-t^2}{2tx} + \frac{x^2}{2tx} = \frac{-t}{2x} + \frac{x}{2t}$

$\frac{dx}{dt} - \frac{x}{2t} = \frac{-t}{2x}$ $n = -1$

$\frac{dx}{dt} + P(t)x = Q(x)x^{-1}$

3) Bernoulli: $\frac{dy}{dx} + \frac{y}{x} = x^3y^2$ $n = 2$

$P(x) = 1/x$ $Q(x) = x^3$

11) $(y^2 - xy)dx + x^2dy = 0$ $\frac{dy}{dx} = \frac{-(y^2-xy)}{x^2} = \frac{-y^2}{x^2} + \frac{y}{x}$

$G(v) = -v^2 + v$

$v + x\frac{dv}{dx} = G(v)$

$\int \frac{1}{v - v^2 - v}\, dv = \int \frac{1}{x}\, dx$

$v^{-1} = \ln x + C$

$\frac{x}{y} = \ln x + C$ $y = \frac{x}{\ln|x| + C}$ $y = 0, x = 0$

13) $\frac{dx}{dt} = \frac{x^2 + \sqrt{t^2+x^2}}{tx}$

$\frac{dx}{dt} = \frac{(x^2/t^2) + \sqrt{1 + x^2/t^2}}{(x/t)}$

$v = x/t \qquad x = vt \qquad \frac{dx}{dt} = v + t\frac{dv}{dt}$

$v + t\frac{dv}{dt} = \frac{v^2 + \sqrt{1+v^2}}{v}$

$\int \frac{v}{\sqrt{1+v^2}}\, dv = \int \frac{1}{t}\, dt$

$\sqrt{1 + x^2/t^2} = \ln|t| + C$

21) $\frac{dy}{dx} + \frac{y}{x} = x^2y^2 \qquad P(x) = 1/x$

$Q(x) = x^2 \qquad n = 2$

$v = y^{1-2} = y^{-1}$

$\frac{1}{1-2}\frac{dv}{dx} + \frac{1}{x}v = x^2$

$\frac{dv}{dx} - \frac{1}{x}v = -x^2$

$p(x) = -1/x \qquad \mu = x^{-1}$

$\frac{d}{dx}\left(\frac{v}{x}\right) = -x$

$\frac{v}{x} = \frac{-x^2}{2} + C \qquad V = \frac{-x^3}{2} + Cx$

$$y^{-1} = -\frac{x^3}{2} + Cx$$

$$y = \frac{2}{Cx - x^3}$$ and $y = 0$ is a solution

28) $$\frac{dy}{dx} + y^2x + y = 0$$

$$\frac{dy}{dx} + y = -xy^3 \qquad p(x) = 1 \qquad Q(x) = -x \qquad n = 3$$

$$\frac{1}{1-3}\frac{dv}{dx} + v = -x$$

$$\frac{dv}{dx} - 2v = 2x \qquad p(x) = -2 \qquad \mu(x) = e^{-2x}$$

$$\frac{d}{dx}\left(e^{-2x}v\right) = 2xe^{-2x}dx$$

$$e^{-2x}v = 2\int xe^{-2x}dx \qquad \begin{array}{ll} u = x & du = dx \\ dv = e^{-2x}dx & v = -e^{-2x}/2 \end{array}$$

$$e^{-2x}v = 2\left(\frac{-xe^{-2x}}{2} + \int \frac{e^{-2x}}{2}dx\right) = 2\left(\frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4} + C\right)$$

$$v = 2e^{2x}\left(\frac{xe^{-2x}}{2} + \frac{e^{-2x}}{4} + C\right) = -x - 1/2 + Ce^{2x}$$

$$y^{-2} = -x - 1/2 + Ce^{2x} \qquad y = \pm\frac{1}{\sqrt{-x - 1/2 + Ce^{2x}}}$$