Partial Fractions

$$\int \frac{dx}{(x-a)^n}$$

1. $n = 1$

$$\int \frac{dx}{(x-a)} = \ln|x-a| + C.$$

2. $n > 1$

$$\int \frac{dx}{(x-a)^n} = \frac{1}{(1-n)} \cdot \frac{1}{(x-a)^{n-1}} + C.$$

$$\int \frac{A+Bx}{([x-a]^2+b^2)^n}\,dx = \int \frac{B[x-a]}{([x-a]^2+b^2)^n}\,dx + \int \frac{A+Ba}{([x-a]^2+b^2)^n}\,dx.$$

1. $n = 1$

$$\int \frac{A+Bx}{[x-a]^2+b^2}\,dx = \frac{B}{2}\ln\left([x-a]^2+b^2\right) + \frac{A+Ba}{b}\tan^{-1}\frac{x-a}{b} + C.$$

2. $n > 1$

$$\int \frac{B[x-a]}{([x-a]^2+b^2)^n}\,dx = \frac{B}{2(1-n)} \cdot \frac{1}{([x-a]^2+b^2)^{n-1}} + C.$$

$$\int \frac{A+Ba}{([x-a]^2+b^2)^n}\,dx = (A+Ba)\int \frac{1}{b^{2n}\left(\left[\frac{x-a}{b}\right]^2+1\right)^n}\,dx.$$

The substitution

$$\frac{x-a}{b} = \tan\theta,\ dx = \frac{bd\theta}{\cos^2\theta},\ \left[\frac{x-a}{b}\right]^2 + 1 = \frac{1}{\cos^2\theta}$$

reduces the integral to $\frac{A+Ba}{b^{2n-1}}\int \cos^{2(n-1)}\theta\ d\theta.$

$$\int \cos^{2(n-1)}\theta\ d\theta = \text{(integration by parts)} = \frac{\cos^{2n-3}\theta\sin\theta}{2(n-1)} + \frac{2n-3}{2n-2}\int \cos^{2(n-2)}\theta\ d\theta + C.$$

So

$$\int \cos^{2(n-1)}\theta\ d\theta = \frac{x-a}{b\left(\left[\frac{x-a}{b}\right]^2+1\right)^{n-1}(2n-2)} + \frac{2n-3}{b(2n-2)}\int \frac{dx}{\left(\left[\frac{x-a}{b}\right]^2+1\right)^{n-1}} + C,$$

and

$$\int \frac{A+Ba}{([x-a]^2+b^2)^n}\,dx = \frac{(A+Ba)(x-a)}{b^2\left([x-a]^2+b^2\right)^{n-1}(2n-2)} + \frac{(A+Ba)(2n-3)}{b^2(2n-2)}\int \frac{dx}{\left([x-a]^2+b^2\right)^{n-1}} + C.$$

Finally

$$\int \frac{A+Bx}{([x-a]^2+b^2)^n}\,dx =$$

$$\frac{B}{2(1-n)([x-a]^2+b^2)^{n-1}} + \frac{(A+Ba)(x-a)}{b^2\left([x-a]^2+b^2\right)^{n-1}(2n-2)} + \frac{(A+Ba)(2n-3)}{b^2(2n-2)}\int \frac{dx}{\left([x-a]^2+b^2\right)^{n-1}} + C.$$