MATH225
quiz #1, 02/26/19
Total 100
Solutions

Show all work legibly. **Name**:______________________

1. (20) Consider $y' = -y^2\cos(2t)$.

   (a) (5) Identify $y' = -y^2\cos(2t)$ as:

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ with linear coefficients
   ▫ none of the above

   (b) (15) Solve $y' = -y^2\cos(2t)$.

   The solution is:
   Solution.

$$\begin{aligned} -\frac{y'}{y^2} &= \cos(2t) \\ \int -\frac{y'}{y^2}\,dy &= \int \cos(2t)\,dt \\ \frac{1}{y} &= \frac{1}{2}\sin(2t) + c \\ y(t) &= \frac{2}{\sin(2t) + 2c} \end{aligned}$$

2. (20) Consider $2tyy' + 2t + y^2 = 0$.

   (a) (5) Identify $2tyy' + 2t + y^2 = 0$ as:

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ with linear coefficients
   ▫ none of the above

   (b) (15) Solve $2tyy' + 2t + y^2 = 0$.

   The solution is: $t^2 + ty^2 = c$.
   Solution. Note that $\dfrac{\partial(2t+y^2)}{\partial y} = 2y = \dfrac{\partial(2ty)}{\partial t}$, hence the equation is an exact equation, and

$$\begin{aligned} F(t,y) &= F(0,0) + \int_0^t F_t(x,0)\,dx + \int_0^y F_y(t,s)\,ds \\ &= F(0,0) + \int_0^t 2x\,dx + \int_0^y 2ts\,ds \\ &= F(0,0) + t^2 + ty^2. \end{aligned}$$

3. (20) Consider $y' = y + 2y^5$.

   (a) (5) Identify $y' = y + 2y^5$ as:

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ with linear coefficients
   ▫ none of the above

   (b) (15) Solve $y' = y + 2y^5$.
   The solution is:
   Solution. If $u = y^{-4}$ then the equation reduces to the linear equation $u' + 4u = -8$. The general solution for $u' + 4u = 0$ is $u(x) = ce^{-4x}$. A particular solution u_p for the equation $u' + 4u = -8$ is $u_p(x) = -2$. The general solution for $u' + 4u = -8$ is $-2 + ce^{-4x}$. Hence $y(x) = \pm\frac{1}{(ce^{-4x} - 2)^{1/4}}$.

4. (20) Consider $y' = 2y + 3$.

   (a) (5) Identify $y' = 2y + 3$ as:

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ with linear coefficients
   ▫ none of the above

   (b) (15) Solve $y' = 2y + 3$, $y(0) = 1$.
   The solution is:
   Solution. First solve $y' = 2y$ with the general solution $y(x) = ce^{2x}$. Next find a particular solution $y_p(x)$ for $y' = 2y + 3$. This might be $y_p(x) = -\frac{3}{2}$. Hence the general solution for $y' = 2y + 3$ is $ce^{2x} - \frac{3}{2}$. We need to select c so that $ce^0 - \frac{3}{2} = 1$, i.e. $c - \frac{3}{2} = 1$, and $c = \frac{5}{2}$. The solution to the initial value problem is $\frac{5}{2}e^{2x} - \frac{3}{2}$.

5. (20) Consider $(x^2 - y^2)\,dx + xy\,dy = 0$

   (a) (5) Identify $(x^2 - y^2)\,dx + xy\,dy = 0$ as:

   ▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ with linear coefficients
   ▫ none of the above

   (b) (15) Solve $(x^2 - y^2)\,dx + xy\,dy = 0$.
   The solution is:
   Solution. The substitution $y = xu$ leads to the separable equation $xuu' = 0$ with the general solution $u^2(x) = 2\ln\left|\frac{c}{x}\right|$. Finally $y^2(x) = 2x^2\ln\left|\frac{c}{x}\right|$.

6. (20) Consider $(3x + 2y + 1)dx + (3x + 2y + 1)dy = 0$.

(a) (5) Identify $(3x+2y+1)dx+(3x+2y+1)dy=0$ as:

▫ separable ▫ linear ▫ exact ▫ homogeneous ▫ Bernoulli ▫ with linear coefficients
▫ none of the above

(b) (15) Solve $(3x+2y+1)dx+(3x+2y-1)dy=0$.

The solution is:

Solution. If $u=3x+2y+1$, then $udu+(5u+6)dy=0$, and $dx=\dfrac{2-u}{6-u}du$. This leads to $c+x=u+4\ln|6-u|$, and finally

$$c=2x+3y+4\ln|5-3x-2y|.$$

or else

(c) (15) Solve $(3x+2y+1)dx+(3x+2y+1)dy=0$.

The solution is: $\dfrac{dy}{dx}=-1$, and $y(x)=-x+c$.

Solution.