MATH225
quiz #2, 03/12/19
Total 100
Solutions

Show all work legibly. **Name**:______________________

1. (40)

   - (10) Find the general solution $y(t)$ for $y'' + 5y' + 6y = 0$.
     Solution. The characteristic equation is $r^2 + 5r + 6 = 0$, or $(r+2)(r+3) = 0$. Finally $y_1(t) = e^{-2t}$, and $y_2(t) = e^{-3t}$, and the general solution is
     $$y(t) = c_1 e^{-2t} + c_2 e^{-3t}.$$
   - (20) Find a particular solution $y_p(t)$ for $y'' + 5y' + 6y = 2e^{-2t}$.
     Solution. $y_p(t) = cte^{-2t}$. Note that $y'(t) = ce^{-2t} - 2cte^{-2t}$, and $y''(t) = -4ce^{-2t} + 4cte^{-2t}$. The equation becomes
     $$(-4ce^{-2t} + 4cte^{-2t}) + 5(ce^{-2t} - 2cte^{-2t}) + 6cte^{-2t} = 2e^{-2t}.$$
     This yields $c = 2$, and $y_p(t) = 2te^{-2t}$.
   - (10) Solve the initial value problem $y'' + 5y' + 6y = 2e^{-2t}$, $y(0) = 1$, $y'(0) = 0$.
     Solution. The general solution for the problem is $y(t) = c_1 e^{-2t} + c_2 e^{-3t} + 2te^{-2t}$. One has $y'(t) = -2c_1 e^{-2t} - 3c_2 e^{-3t} + 2e^{-2t} - 6te^{-2t}$. Hence
     $$1 = y(0) = c_1 + c_2, \text{ and } 0 = y'(0) = -2c_1 - 3c_2 + 2.$$
     This yields $c_1 = 1,\ c_2 = 0,$ and $y(t) = e^{-2t} + 2te^{-2t}$.

2. (40)

   - (10) Find the general solution $y(t)$ for $9y'' + 6y' + y = 0$.
     Solution. The characteristic equation is $0 = 9r^2 + 6r + 1 = 9\left(r + \frac{1}{3}\right)^2$. The general solution is $y(t) = c_1 e^{-1/3t} + tc_2 e^{-1/3t}$.
   - (10) Solve the initial value problem $9y'' + 6y' + y = 0,\ y(0) = 1,\ y'(0) = 1$.
     Solution.
     $$1 = y(0) = c_1,\ 1 = y'(0) = -\frac{1}{3}c_1 + c_2, \quad \text{and } c_1 = 1,\ c_2 = \frac{4}{3}.$$
     $y(t) = e^{-1/3t} + \frac{4}{3}te^{-1/3t}$.

- (20) Find a particular solution $y_p(t)$ for $9y'' + 6y' + y = 2t$.
  Solution. $y_p(t) = at + b$, $y_p'(t) = a$, $y_p''(t) = 0$, and $6a + at + b = 2t$. Finally $a = 2$, $b = -12$, and $y_p(t) = 2t - 12$.

3. (40)

- (10) Find the general solution $y(t)$ for $y'' - 2y' + y = 0$.
  Solution. $0 = r^2 - 2r + 1 = (r-1)^2$. The general solution is

$$y(t) = c_1 y_1(t) + c_2 y_2(t) = c_1 e^t + c_2 t e^t.$$

- (10) Solve $y'' - 2y' + y = 0$, $y(0) = y'(0) = 0$.
  Solution. $y(t) = 0$.

- (20) Find a particular solution $y_p(t)$ for $y'' - 2y' + y = \frac{2e^t}{1+t^2}$.
  [Hint: $\int \frac{1}{1+t^2}\,dt = \tan^{-1} t$]
  Solution. $y_p(t) = c_1(t)e^t + c_2(t)te^t$, where

$$c_1'(t) = -\frac{2e^t}{1+t^2} te^t / W(t) = -\frac{2t}{1+t^2}, \text{ and } c_2'(t) = \frac{2e^t}{1+t^2} e^t / W(t) = \frac{2}{1+t^2}$$

with

$$W(t) = y_1(t)y_2'(t) - y_2(t)y_1'(t) = e^t(e^t + te^t) - te^t e^t = e^{2t}.$$

Hence

$$c_1(t) = -\ln(1+t^2), \text{ and } c_2(t) = 2\tan^{-1}(t).$$

Finally $y_p(t) = -\ln(1+t^2)e^t + 2te^t \tan^{-1} t$.