MATH225

quiz #3, 04/09/19

Total 100

Solutions

Show all work legibly. **Name**:________________________

1. (40)

   (a) (20) Find the inverse Laplace transform $y(t)$ of $Y(s) = \frac{3}{(s-1)^2(s-2)}$

   $y(t) =$

   Solution.

$$\frac{3}{(s-1)^2(s-2)} = \frac{A}{s-2} + \frac{Bs+C}{(s-1)^2}, \text{ with } A = 3,\ B = -3,\ C = 0.$$

   Hence

$$Y(s) = \frac{3}{s-2} - \frac{3}{s-1} - \frac{3}{(s-1)^2}, \text{ and } y(t) = 3\left(e^{2t} - e^t - te^t\right).$$

   (b) (20) Use the Laplace transform method to solve $y'' - 4y' + 4y = 3e^t$, $y(0) = y'(0) = 0$.

   $y(t) =$

   Solution. The Laplace transform of the equation leads to

$$(s^2 - 4s + 4)Y(s) = \frac{3}{s-1}, \text{ and } Y(s) = \frac{3}{(s-1)(s-2)^2}.$$

   Hence

$$Y(s) = \frac{3}{s-1} + \frac{-3s+9}{(s-2)^2} = \frac{3}{s-1} - \frac{3}{s-2} + \frac{3}{(s-2)^2}$$

   and $y(t) = 3\left(e^t - e^{2t} + te^{2t}\right)$.

2. (30) Find the inverse Laplace transform $y(t)$ of the function $Y(s) = \frac{2s-3}{9s^2-12s+20}$.

   $y(t) =$

   Solution. Note that

$$\begin{aligned} Y(s) &= \frac{2s-3}{9s^2-12s+20} = \frac{2s-3}{(3s-2)^2+16} = \frac{2s-3}{9[(s-2/3)^2+16/9]} \\ &= \frac{2}{9}\frac{s-2/3}{(s-2/3)^2+(4/3)^2} - \frac{5}{36}\frac{4/3}{(s-2/3)^2+(4/3)^2}, \end{aligned}$$

   and $y(t) = \frac{2}{9}e^{2t/3}\cos\frac{4t}{3} - \frac{5}{36}e^{2t/3}\sin\frac{4t}{3}$.

3. (30) Solve the initial value problem $y''(t) + 4y(t) = \cos t$, $y(0) = y'(0) = 0$.

$y(t) =$

Solution. We apply the Laplace transform to both sides of the equation

$$Y(s) = \frac{s}{(s^2+1)(s^2+4)} = \frac{1}{3}\frac{s}{(s^2+1)} - \frac{1}{3}\frac{s}{(s^2+4)}.$$

Finally $y(t) = \frac{1}{3}\cos t - \frac{1}{3}\cos(2t)$.

4. (20) Use the Laplace transform to solve $y' + 2y = 0$, $y(0) = 3$.

$y(t) =$

Solution. We apply the Laplace transform to both sides of the equation and obtain

$$(s+2)Y(s) = 3, \text{ and } Y(s) = \frac{3}{s+2}.$$

From the Laplace transform table $L\{e^{-2t}\}(s) = \frac{1}{s+2}$, and finally $y(t) = 3e^{-2t}$.