February 14, 2019 Homework 2 due February 21, 2019

1. Show that for each $n \in \mathbf{N}$ which is different from 0 there is $m \in \mathbf{N}$ such that $S(m) = n$.

2. Let a, b, c, and $d \in \mathbf{N}$. Show that if $a \geq b$ and $c \geq d$, then $a + c \geq b + d$.

3. Let $\mathbf{N} = \{0, 1\}$ and define $S(0) = 1$, and $S(1) = 1$. Show that $\mathbf{N}$ satisfies Peano's axioms 1 and 3, but not 2. Let $\psi : \mathbf{N} \to \mathbf{N}$ such that $\psi(0) = 1$, and $\psi(1) = 0$. Show that the recursion theorem breaks down for this ψ. That is there exists no map $f : \mathbf{N} \to \mathbf{N}$ such that $f(0) = 0$, and $f(S(n)) = \psi(f(n))$.

4. If x, y, and $z \in \mathbf{Z}$ show that

   a) $(x + y)z = xz + yz$,

   b) if $z \neq 0$, then $xz = yz$ implies $x = y$.

   c) if $x \geq y$ and $z \geq 0$, then $xz \geq yz$.

   d) if $x \geq y$, then $x + z \geq y + z$.

   e) If $x \neq y$, then $x > y$ or $y > x$.