February 28, 2019 Homework 3 due March 7, 2019

1. Let $a > b \geq 0$ be integers, and $a = bq + r$, $0 \leq r < b$. Show that $\gcd(a, b) = \gcd(b, r)$.

2. Desscribe all positive integers n so that $gcd(n, n + 2) = 2$.

3. Let $a, b, c \in \mathbf{N}$, and $d' = \min\{ax + by + cz > 0 \ : \ x, y, z \in \mathbf{Z}\}$. Show that $d|a$, $d|b$, and $d|c$, and for each divisor D of a, b and c one has $D|d$.

4. Let p be a prime number. If $\gcd(a, p) = 1$, then $\gcd(a^2, p) = 1$.

5. True or False? $a^m - 1$ is a composite when $a > 2$.

6. Let p be a prime. True or False? $p^m + 1$ is a composite.

7. Let $F_n = 2^{2^n} + 1$. That is $F_0 = 3$, $F_1 = 5$, $F_2 = 17$.

   a) Show that $\displaystyle\prod_{k=0}^{n-1} F_k = F_n - 2$.

   b) Based on the result above can you conclude that F_k and F_n are relatively prime when $k \neq n$?