

March 7, 2019 Homework 4 due March 14, 2019

1. True or False?  $n^2 + 1$  is not divisible by 11 for each  $n \in \mathbb{Z}$ .
2.  $21(n^2 + 1)$  is not divisible by 11 for each  $n \in \mathbb{Z}$ .
3. Let  $n = 8k + 7$ . True or False? There are integers  $a$ ,  $b$ , and  $c$  so that  $n = a^2 + b^2 + c^2$ .
4. Solve  $x^4 + x^3 + x^2 + x + 1 \equiv 0 \pmod{2}$ .
5. True or False? If  $p$  is a prime, and  $a, b > 1$ , then  $(a + b)^p \equiv a^p + b^p \pmod{p}$ .
6. Find all integers  $n$  such that  $3n + 7$  is divisible by 11.
7. Show that  $10^{2n} = 11q + 1$ , and  $10^{2n+1} = 11q - 1$ .
8. Consider a  $k$  digit integer  $n = n_{k-1} \dots n_1 n_0$ . True or False? If  $\sum_{i \text{ is even}} n_i \equiv \sum_{i \text{ is odd}} n_i \pmod{11}$ , then  $11|n$ .