March 28, 2019 Homework 6 due April 11, 2019

1. Let a, n be positive integers with $gcd(a, n) = 1$. Show that there is an integer k such that $a \cdot a^k \equiv 1 \pmod{n}$.

2. If $gcd(n, m) = 1$, then $\varphi(n)\varphi(m) = \varphi(nm)$.

3. Show that if $n > 2$, then $\varphi(n)$ is even.

4. Let n be a positive integer with no square factors (except 1). Show that for each $0 < a < n$ and $1 \leq k$ one has $[a]_n^k \neq [0]_n$.

5. True or False? If $a|b$, then $\varphi(a)|\varphi(b)$.

6. True or False? If $b = ac$, then $\varphi(b) = \varphi(a)\varphi(c)$.

7. Compute $\displaystyle\sum_{d|n} \varphi(d)$ for $n = 12$ and $n = 18$.

8. What can be concluded based on results of Problem 7?