April 11, 2019 Homework 7 due April 25, 2019

1. True or False? If $\sigma, \tau \in S_n$, then $\sigma\tau = \tau\sigma$.

2. If s_1, s_2, s_3, and s_4 are distinct, then

$$(s_2, s_3)(s_1, s_4) = (s_1, s_4)(s_2, s_3), \text{ and } (s_2, s_3)(s_1, s_3) = (s_1, s_2)(s_2, s_3).$$

3. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$. Represent σ as a product of transpositions.

4. Note that $(1, 2)$ is an odd permutation. Indeed $(1, 2)(1, 2) = (1)$. If σ is a cycle of length 3, and $\sigma \in S_3$, then $\tau = (3, \sigma(3))\sigma$ is a cycle of length 2, hence τ is an even permutation. This shows that σ is an odd permutation. Show that

   a) if $\sigma \in S_{2n}$ is a cycle of length $2n$, then σ is an even permutation.

   b) if $\sigma \in S_{2n+1}$ is a cycle of length $2n + 1$, then σ is an odd permutation.

5. Let $k \leq n$. If $(1, \ldots, k)$ is a cycle of length k in S_n, and σ is a cycle of length k in S_n, then there is a transposition τ so that $\tau\sigma\tau^{-1} = (1, \ldots, k)$.

6. Let $k \leq n$, and $\sigma_n \in S_n$ such that $\sigma_n(i) = i$ when $i > k$. We shall say that σ_n is associated with $\sigma_k \in S_k$ in the obvious way if $\sigma_k(i) = \sigma_n(i)$, $i = 1, \ldots, k$. For σ_n, $\tau_n \in S_n$ define $\sigma_n \sim \tau_n$ if $\sigma_n\tau_n^{-1} \in S_k$. For a given $\sigma_n \in S_n$ describe all $\tau_n \in S_n$ so that $\sigma_n\tau_n^{-1} \in S_k$.